APS 425 – Fall 2015

Time Series Analysis: ARIMA Models

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Topics

- Typical time series plot
- Pattern recognition in auto and partial autocorrelations
- Stationarity & invertibility

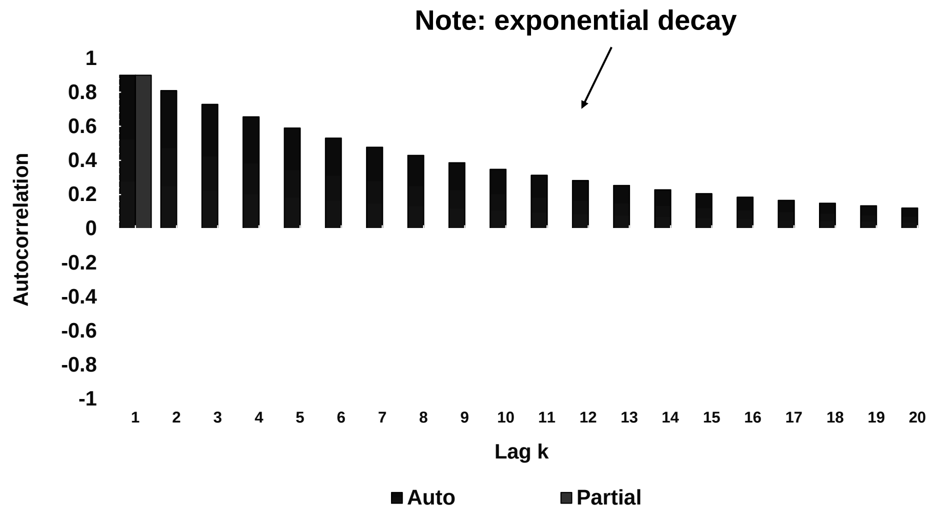
Partial autocorrelations

- Used to identify pure AR models
- Estimate a sequence of AR(k) models, and report the last coefficient estimate, ϕ_{kk}, for each lag k:

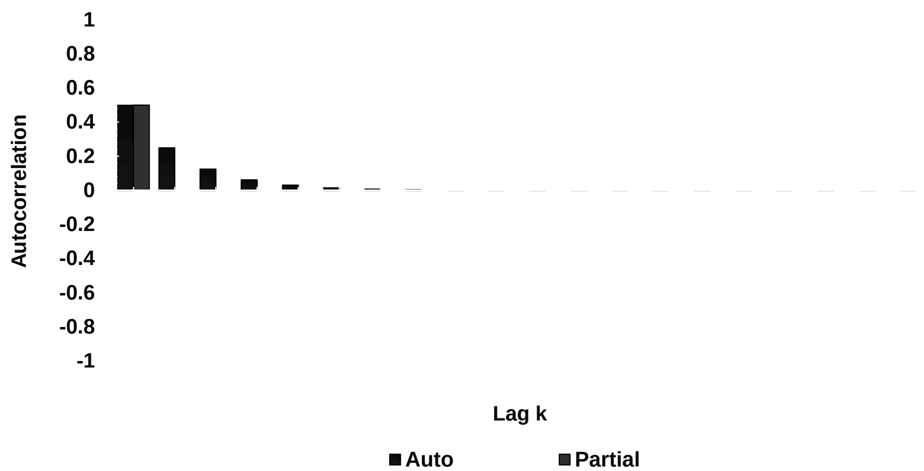
AR(k): $Z_t = \alpha + \phi_{1k} Z_{t-1} + \ldots + \phi_{kk} Z_{t-k} + a_t$

- Graph the pacf coefficients, ϕ_{kk}, and see where they become zero, which implies that the right model is a (k-1)th order AR process

AR(1): $Z_t = \alpha + \phi_1 Z_{t-1} + a_t$
$\phi_1 = .9$



AR(1): $Z_t = \alpha + \phi_1 Z_{t-1} + a_t$
$\phi_1 = .5$



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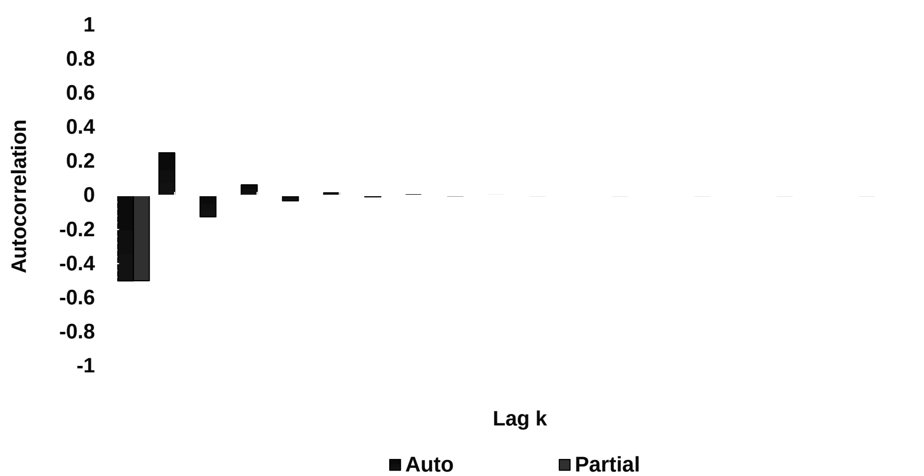

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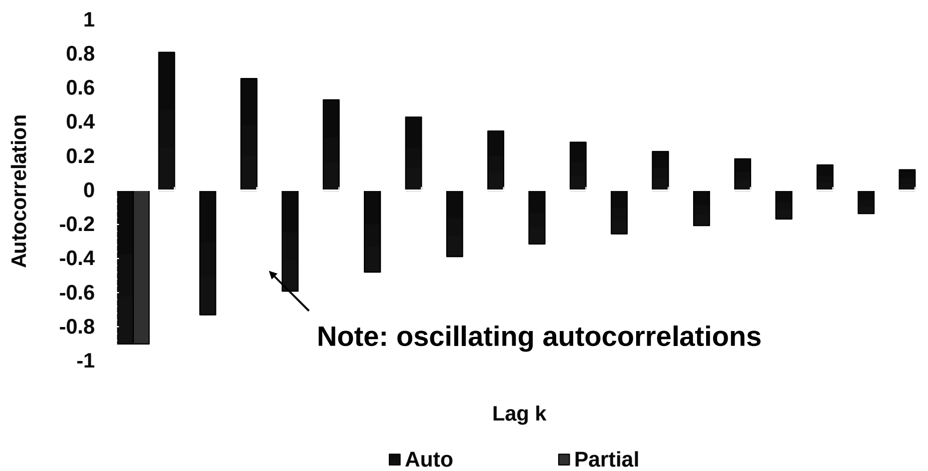

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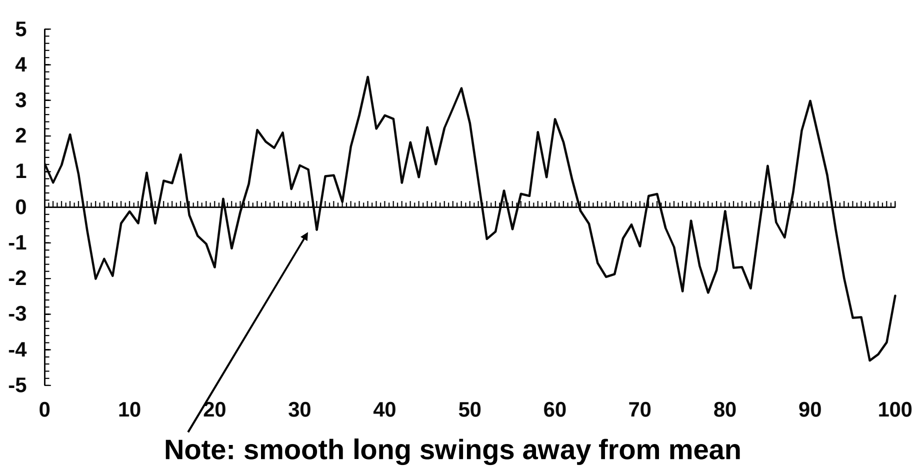

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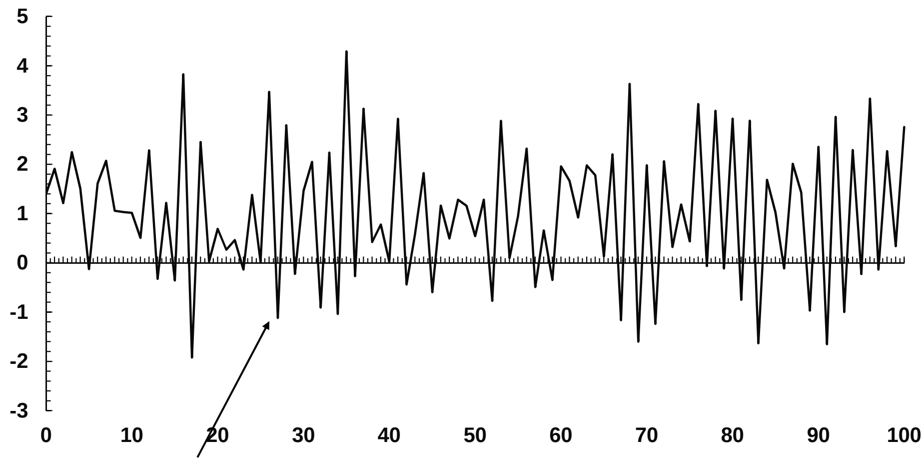
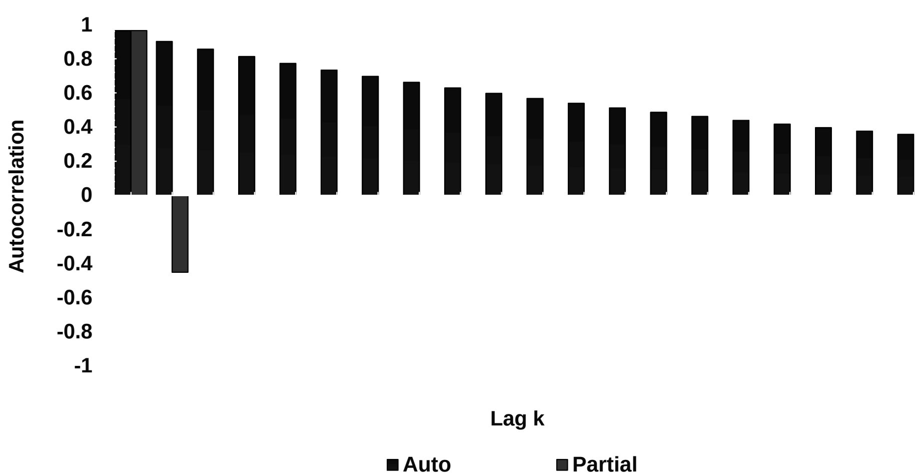


Note: jagged, frequent swings around mean

AR(2): $Z_t = \alpha + \phi_1 Z_{t-1} + \phi_2 Z_{t-2} + a_t$
$\phi_1 = 1.4 \quad \phi_2 = -.45$



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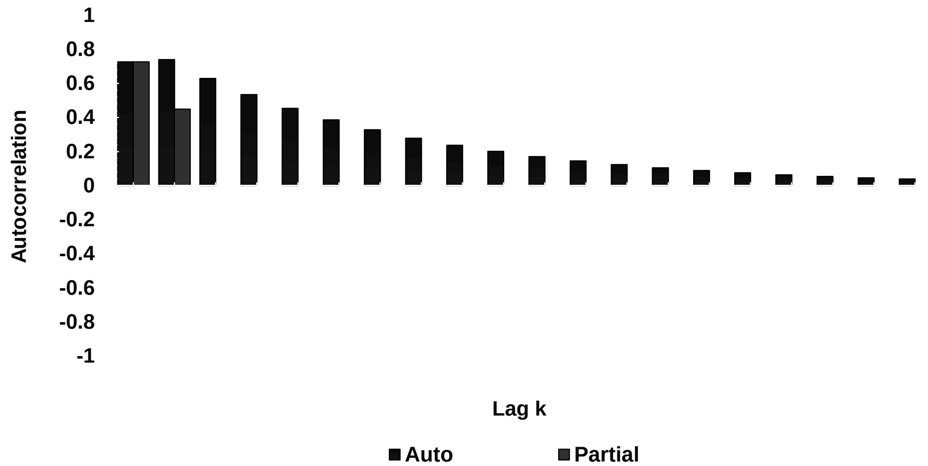

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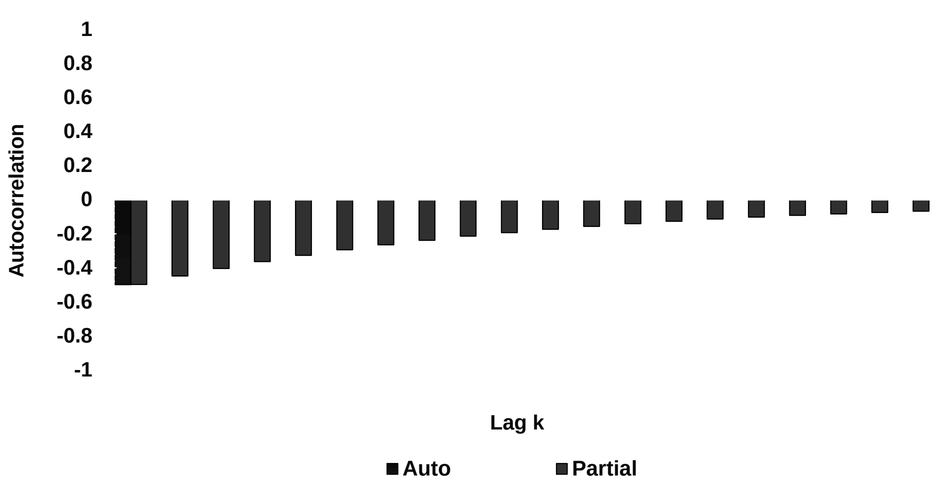
Autoregressive Models: Summary

1) Autocorrelations decay or oscillate

2) Partial Autocorrelations cut-off after lag p, for AR(p) model

3) Stationarity is a big issue
   - very slow decay in autocorrelations
   - should you difference?

MA(1): $Z_t = \alpha + a_t - \theta_1 a_{t-1}$
$\theta_1 = .9$



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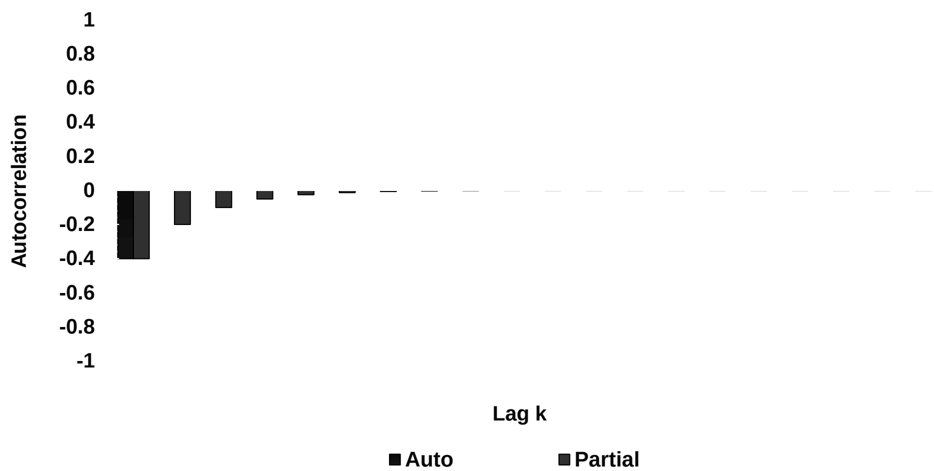

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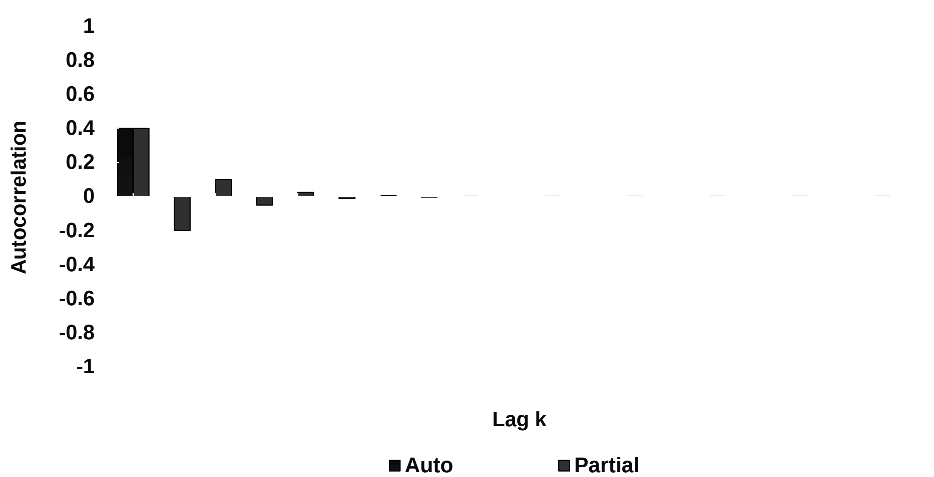

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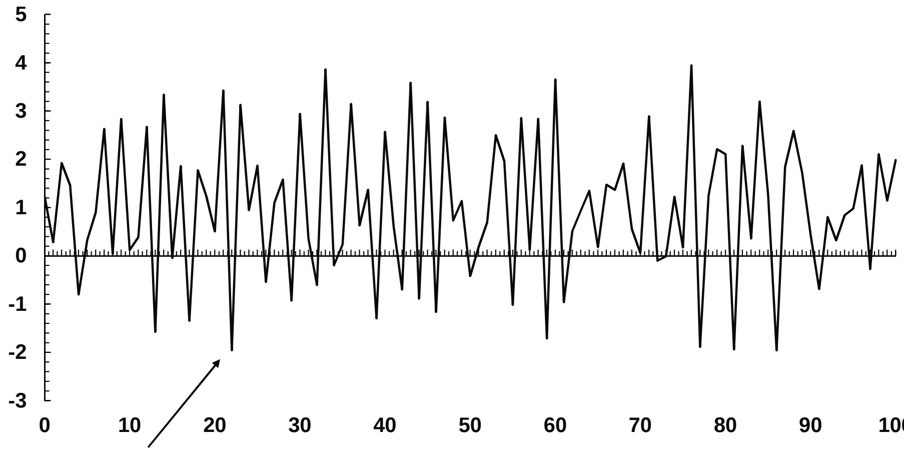


Note: jagged, frequent swings around the mean

Moving Average Models: Summary

1) Autocorrelations cut off after lag q for MA(q) model

2) Partial autocorrelations decay or oscillate
   - because an MA model is equivalent to an infinite order AR process

3) Note that Eviews uses a different convention for the sign of MA coefficients
   - Estimate of MA coefficient is -.75 for IMA(1,1) model of inflation, in BJ notation it would be .75

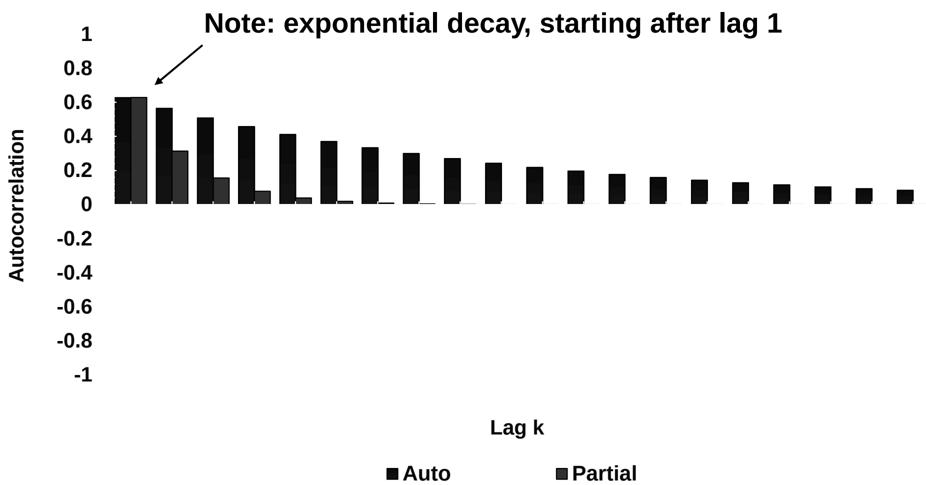
Autoregressive Moving Average Models

ARMA(p,q): $Z_t = \alpha + \phi_1 Z_{t-1} + \ldots + \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \ldots - \theta_q a_{t-q}$

Combines both AR & MA characteristics:

- equivalent to infinite order MA process
  - if stationary
- equivalent to infinite order AR process
  - if "invertible"

ARMA(1,1): $Z_t = \alpha + \phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1}$
$\phi_1 = .9$, $\theta_1 = .5$



ARMA(1,1): $Z_t = \alpha + \phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1}$
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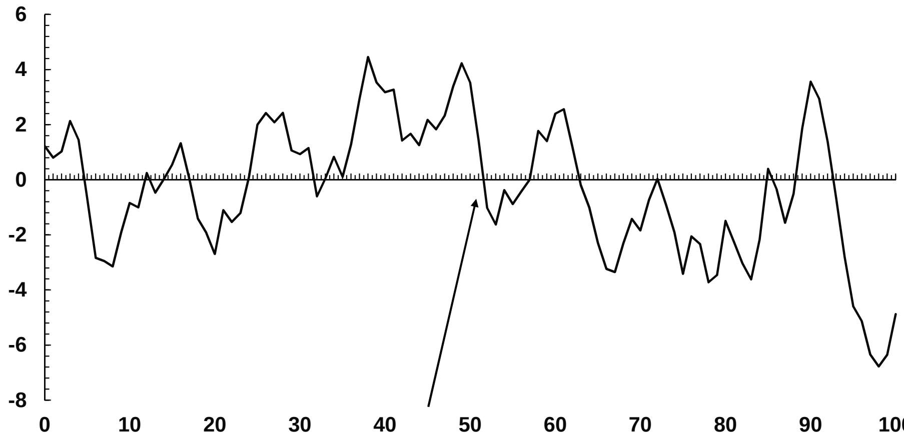


Note: smooth long swings around the mean

Autoregressive Moving Average Models: Summary

1) Autocorrelations decay or oscillate

- because an AR model is equivalent to an infinite order MA process

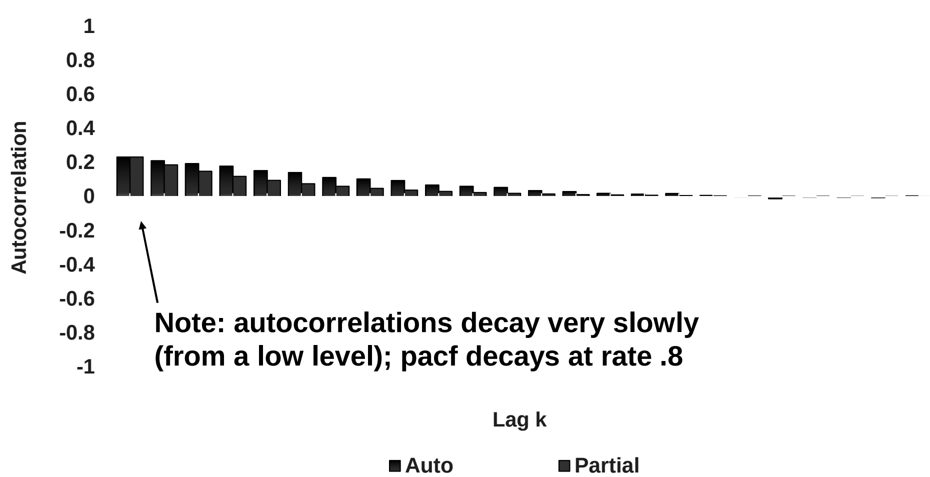
2) Partial Autocorrelations decay or oscillate

- because an MA model is equivalent to an infinite order AR process

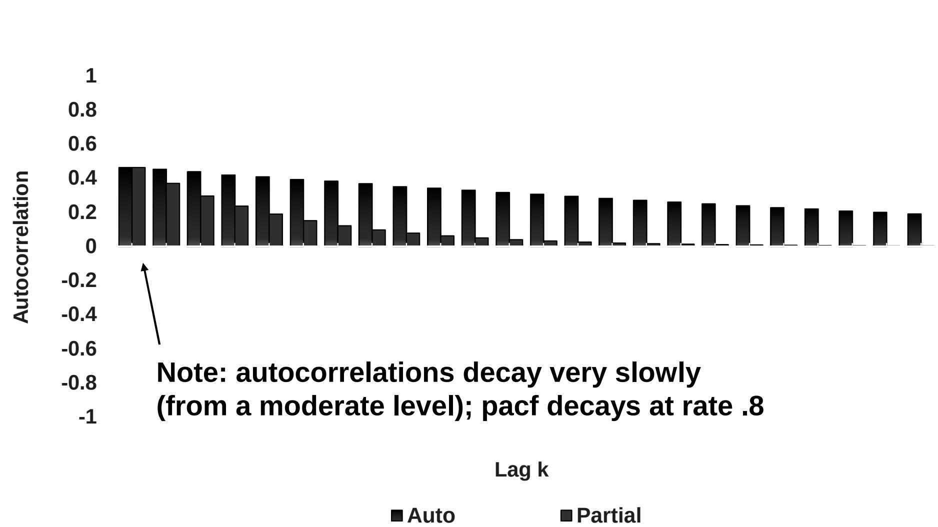
Autoregressive Integrated Moving Average ARIMA(p,d,q) Models

1) ARMA model in the d^{th} differences of the data
2) First step is to find the level of differencing necessary
3) Next steps are to find the appropriate ARMA model for the differenced data
4) Need to avoid "overdifferencing"

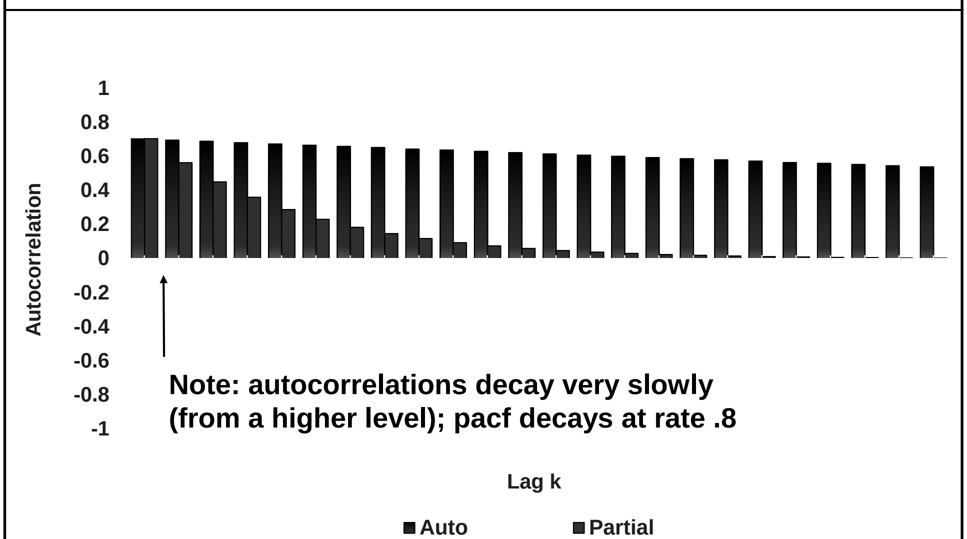
ARIMA(0,1,1): $(Z_t - Z_{t-1}) = a_t - .8\, a_{t-1}$ [T= 50]



ARIMA(0,1,1): $(Z_t - Z_{t-1}) = a_t - .8\ a_{t-1}$ [T=150]



ARIMA(0,1,1): $(Z_t - Z_{t-1}) = a_t - .5\ a_{t-1}$ [T=450]

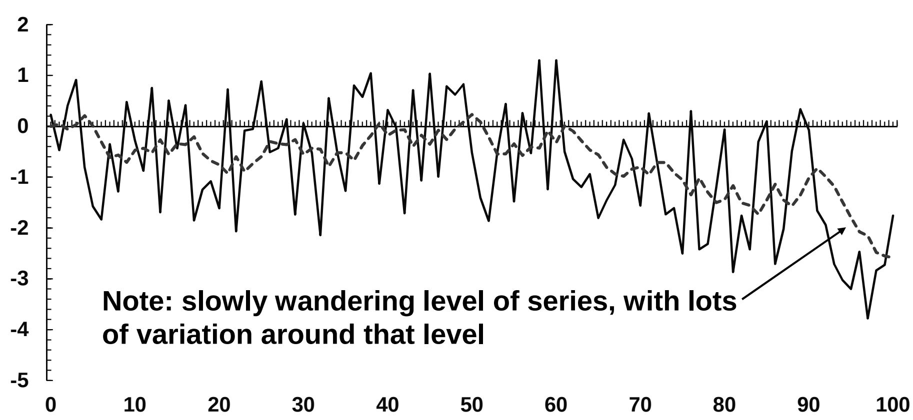


"Autocorrelations" for Nonstationary ARIMA Models

The sample autocorrelations decay very slowly and the level of the autocorrelations is dependent on the length of the sample (in this example T = 50, 150, or 450)

- This would not be true if the series was stationary

ARIMA(0,1,1)
$(Z_t - Z_{t-1}) = a_t - .8\ a_{t-1}$



Seasonal ARIMA(P,D,Q)$_s$ Models

Just like ARIMA(p,d,q) models, except the gap between observations that affect one another is s periods, not 1 period

Example: ARIMA(1,0,0)$_{12}$ model [monthly AR(1)]

$$Z_t = \alpha + \phi_1 Z_{t-12} + e_t$$

- so the autocorrelations decay exponentially at lags 12, 24, 36, etc.
- partial autocorrelation at lag 12 = ϕ_1
  - after lag 12, they equal 0

Seasonal ARIMA(0,1,1)$_s$ Models

This model occurs a lot in real data

- note that seasonal differencing removes a linear trend
- it also removes different fixed means
  - i.e., dummy variables
- if the MA parameter is close to 1, this "exponential smoothing" model implies a slowly wandering level for the series that is different for each period of the year

Seasonal Exponential Smoothing Forecasts

ARIMA$(0,1,1)_{12}$ model:

$$[Z_t - Z_{t-12}] = e_t - \theta_1 e_{t-12}$$

$$\hat{Z}_{t-12}(12) = Z_{t-12} - \theta_1 e_{t-12} = Z_{t-12} - \theta_1 [Z_{t-12} - \hat{Z}_{t-24}(12)]$$

$$= (1 - \theta_1) Z_{t-12} + \theta_1 \hat{Z}_{t-24}(12)$$

- so the forecast for next January is a weighted average of the most recent observation and the forecast for that observation (for Januarys)

Integrated Moving Average Models: Summary

1) Autocorrelations decay slowly
   - initial level is determined by how close MA parameter is to one

2) Partial Autocorrelations decay or oscillate
   - determined by MA parameter

Links

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