

APS 425 – Winter 2008

Time Series Analysis:
ARIMA Models

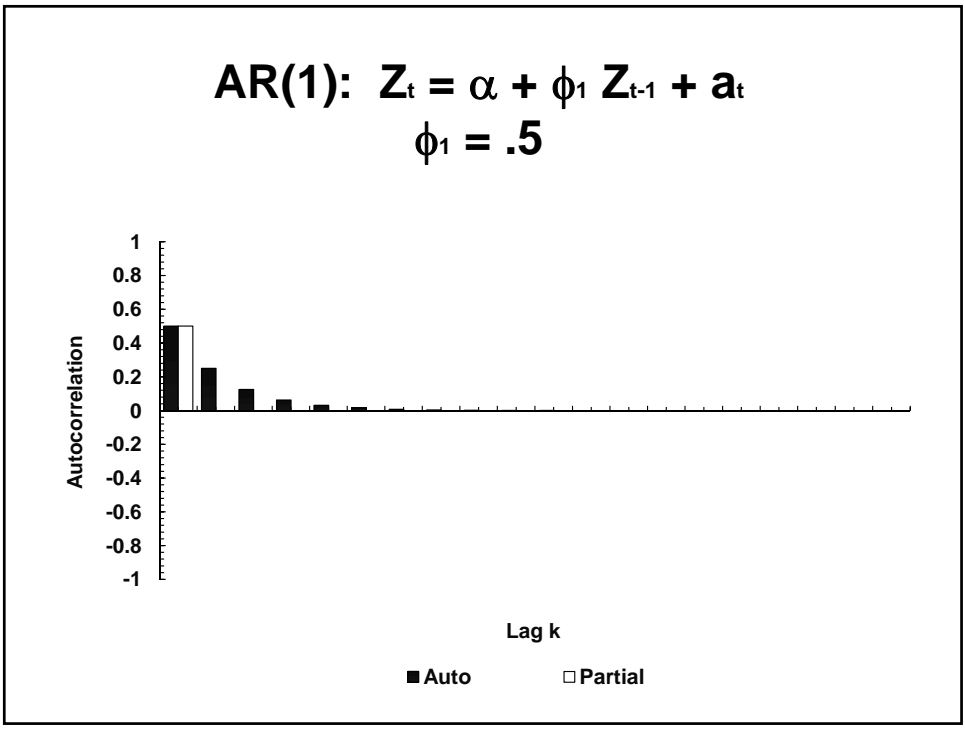
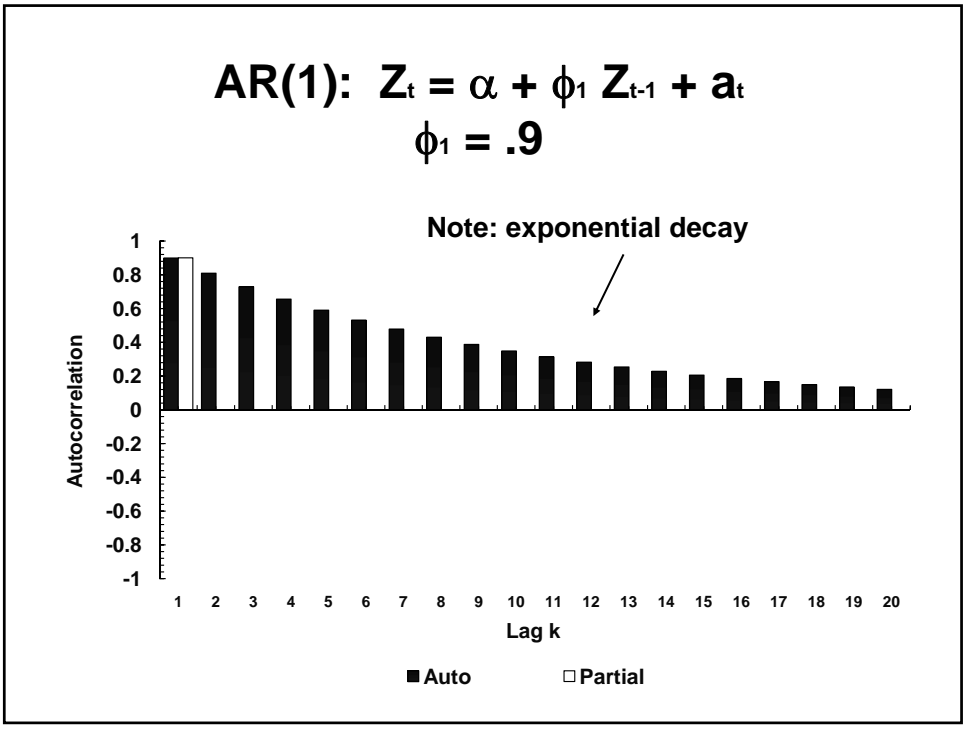
Instructor: G. William Schwert

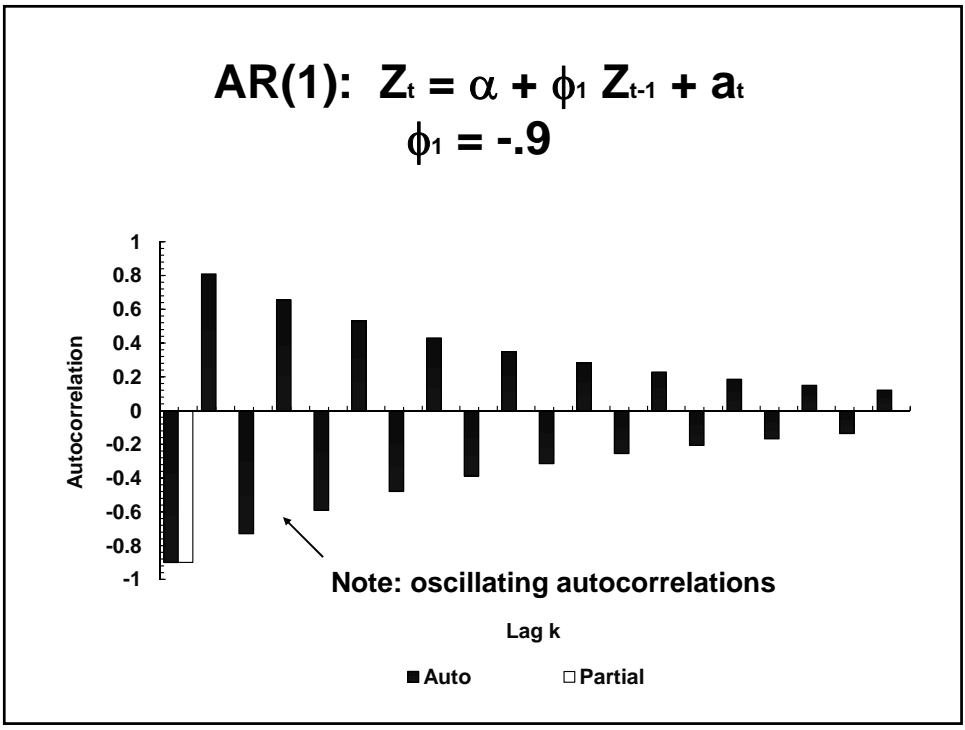
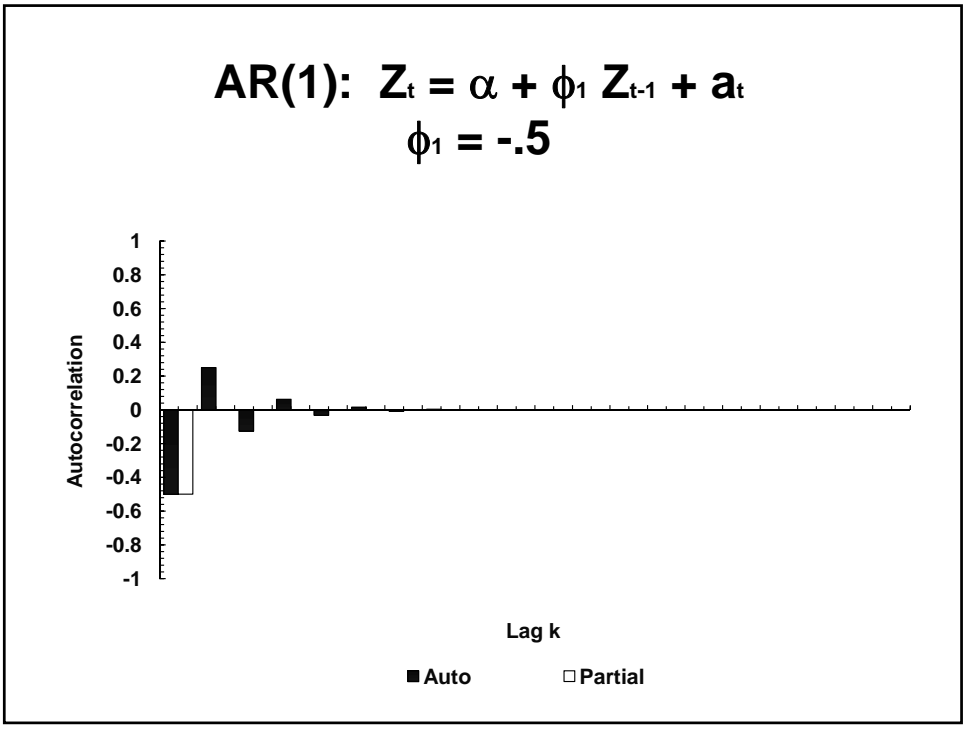
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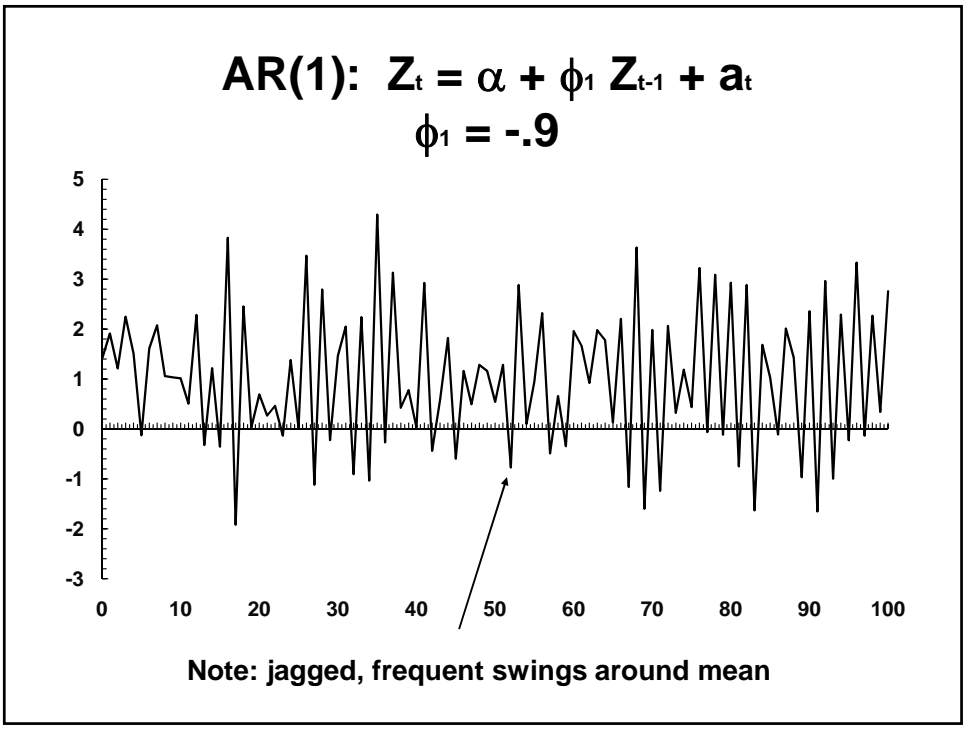
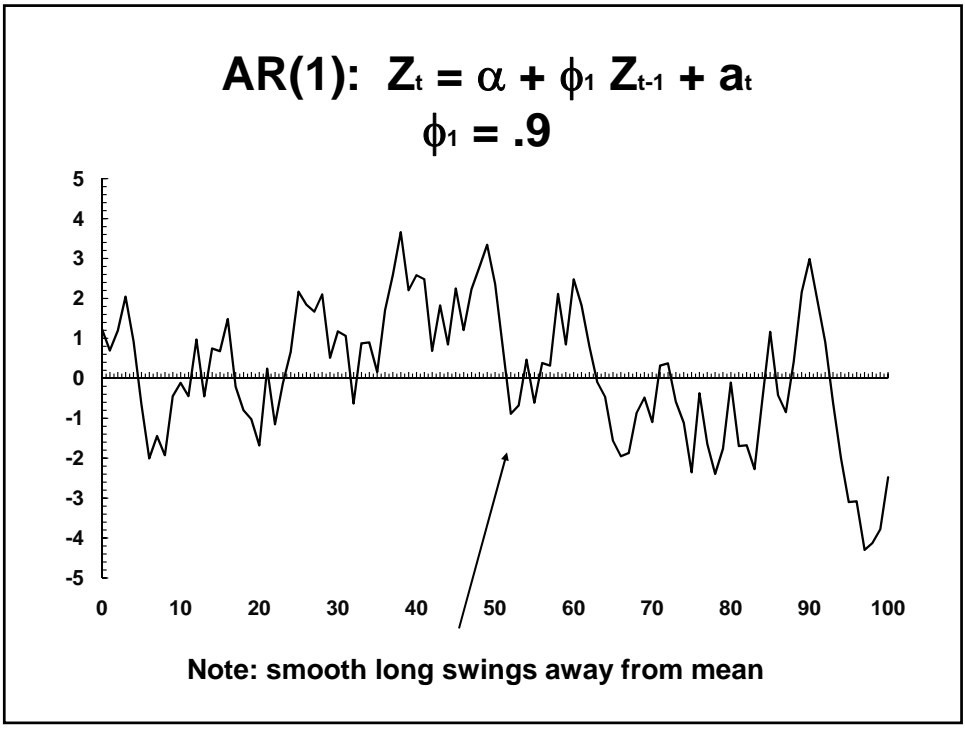
schwert@schwert.simon.rochester.edu

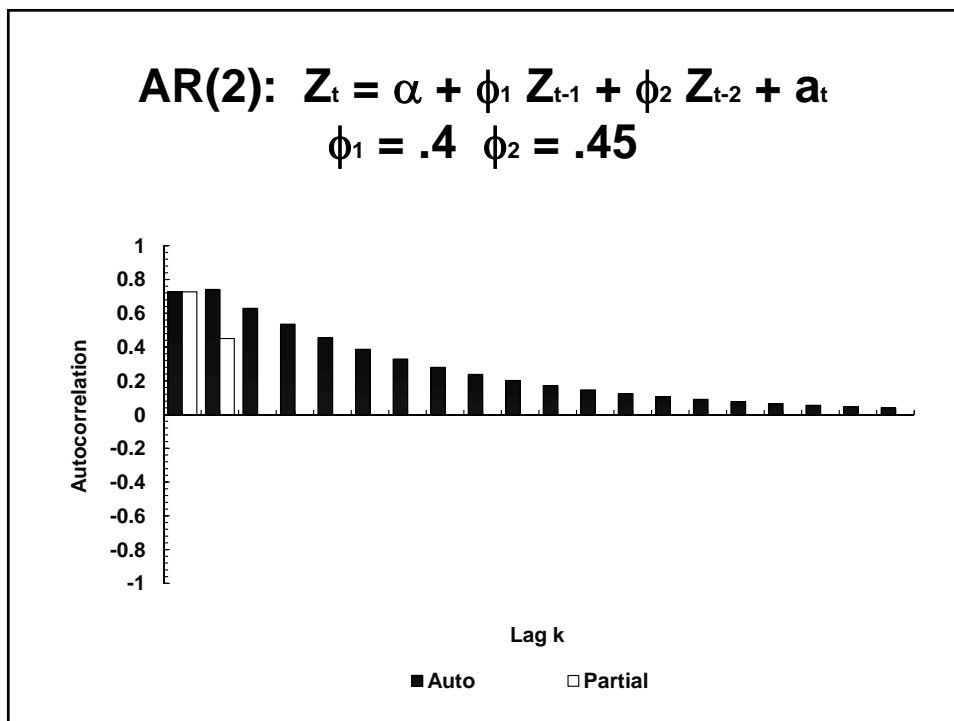
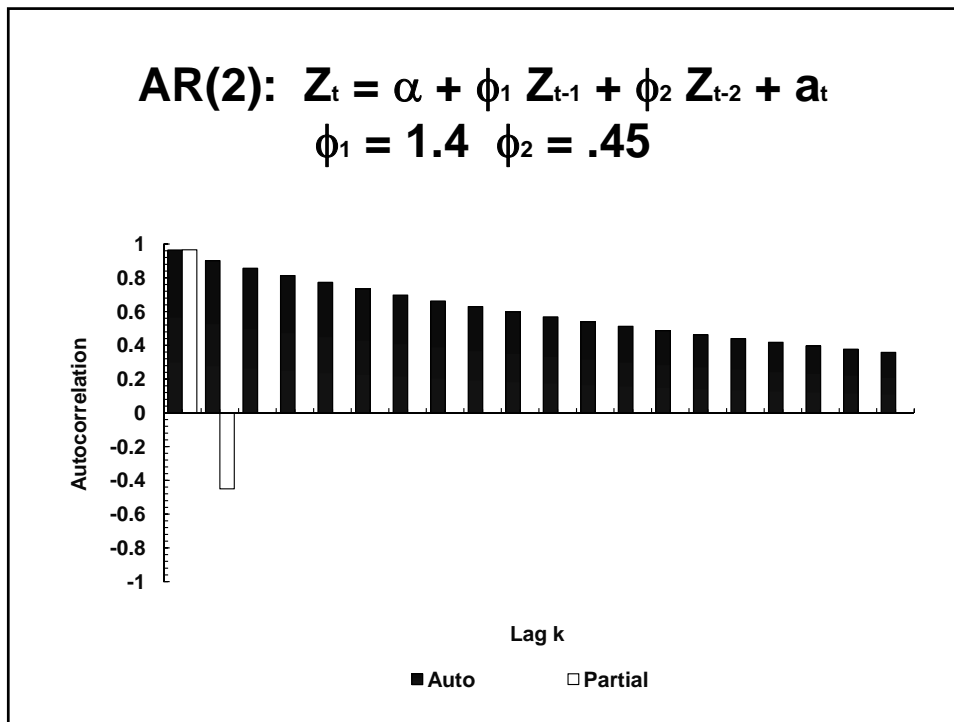
Topics

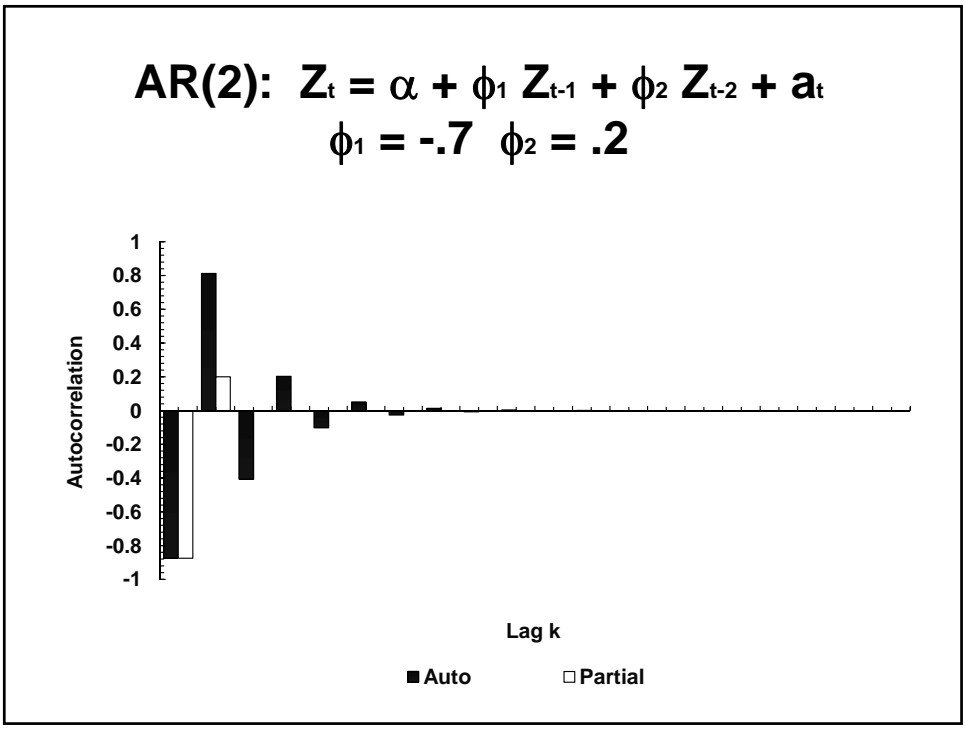
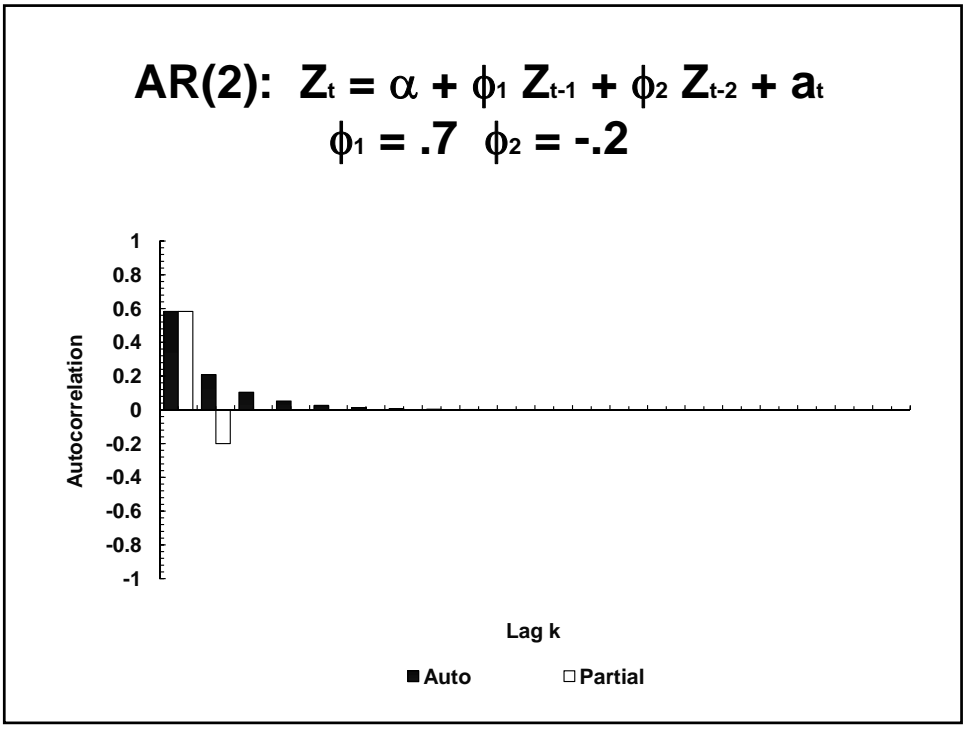
- Typical time series plot
- Pattern recognition in auto and partial autocorrelations
- Stationarity & invertibility

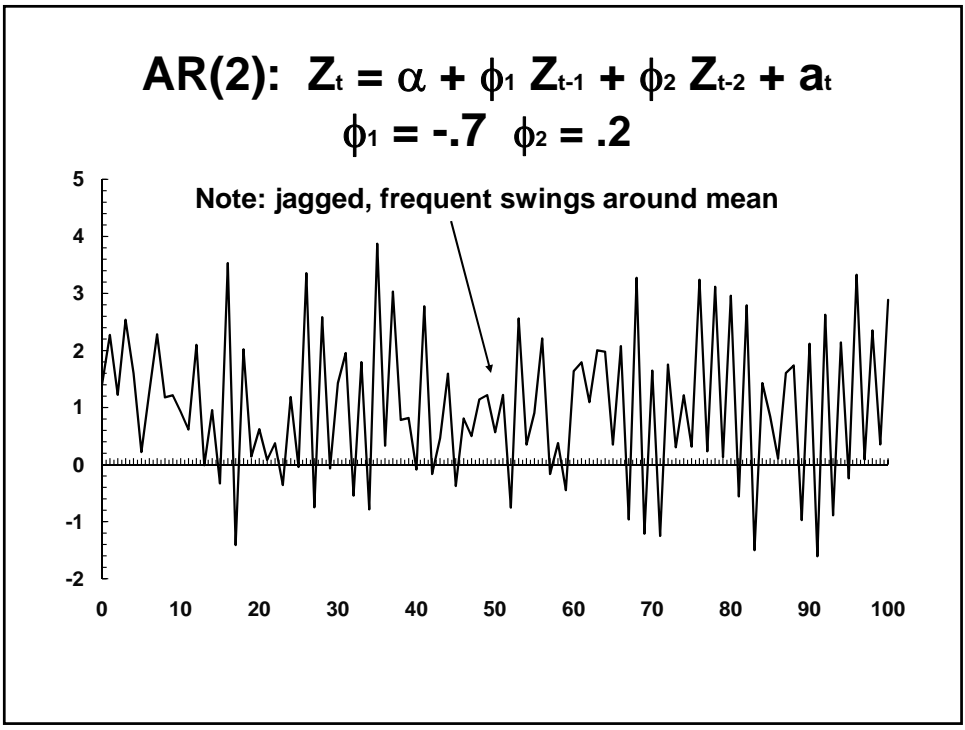
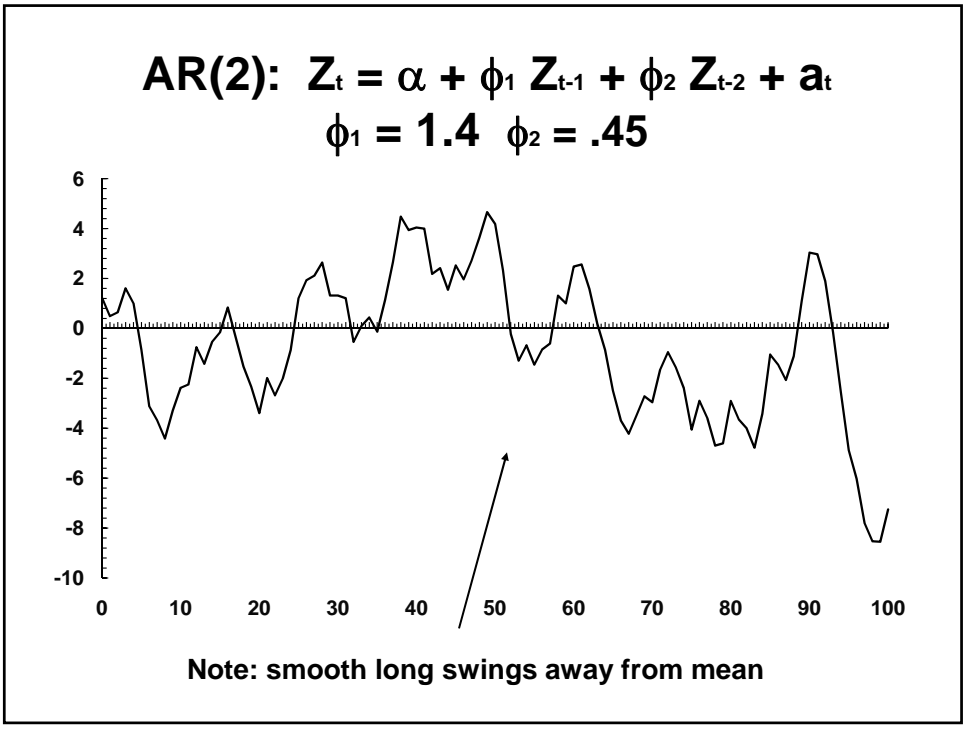










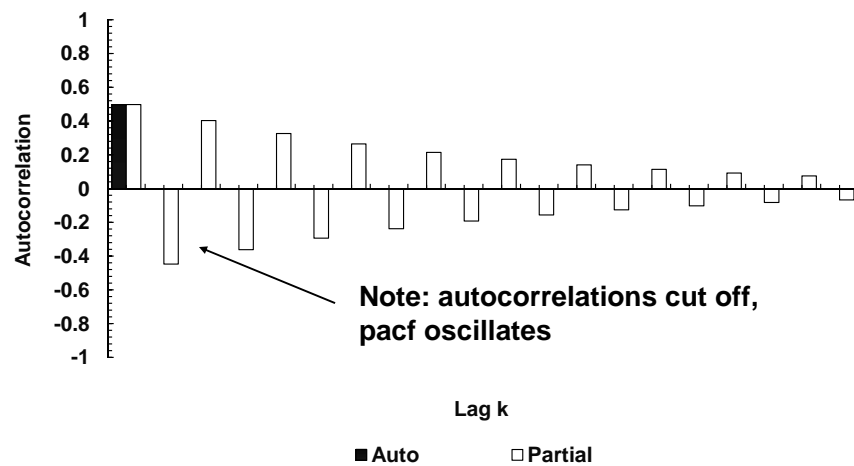


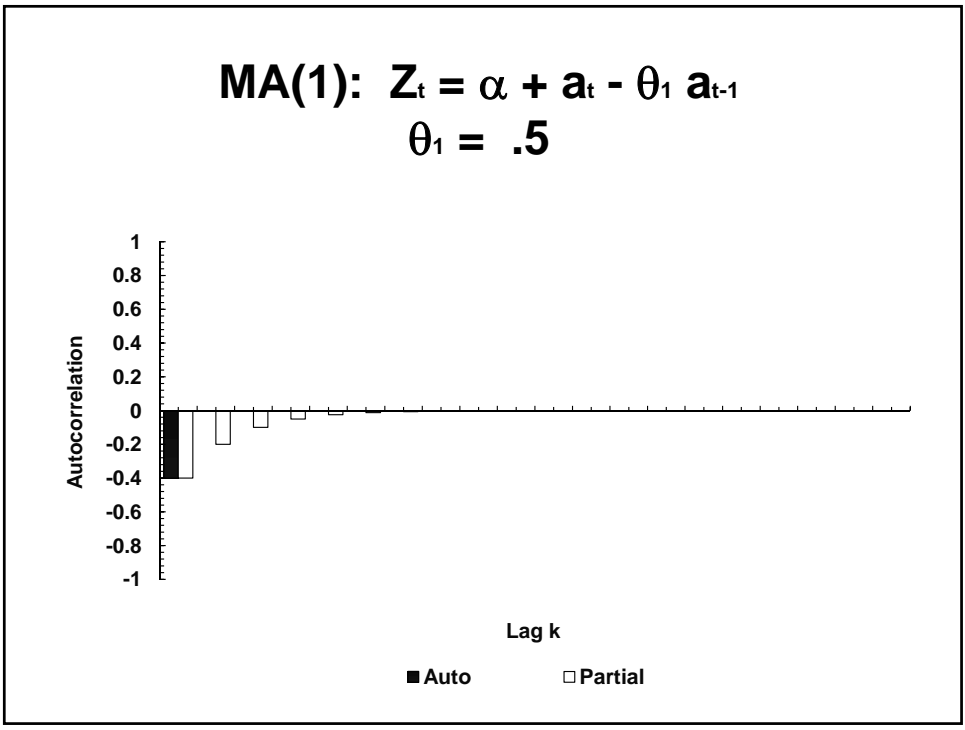
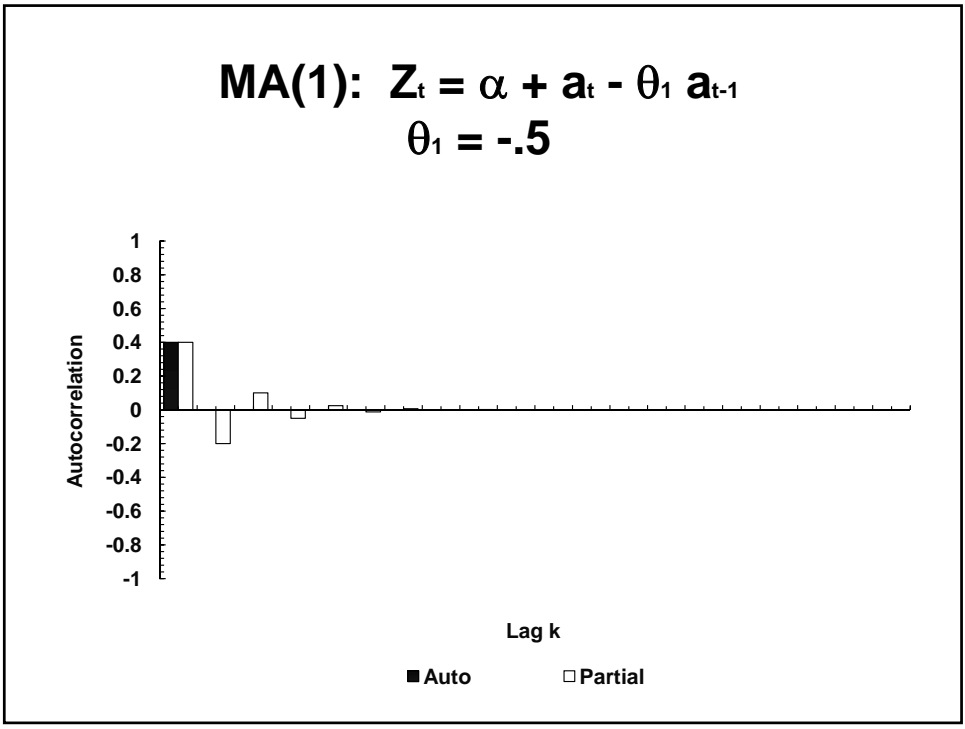
Autoregressive Models: Summary

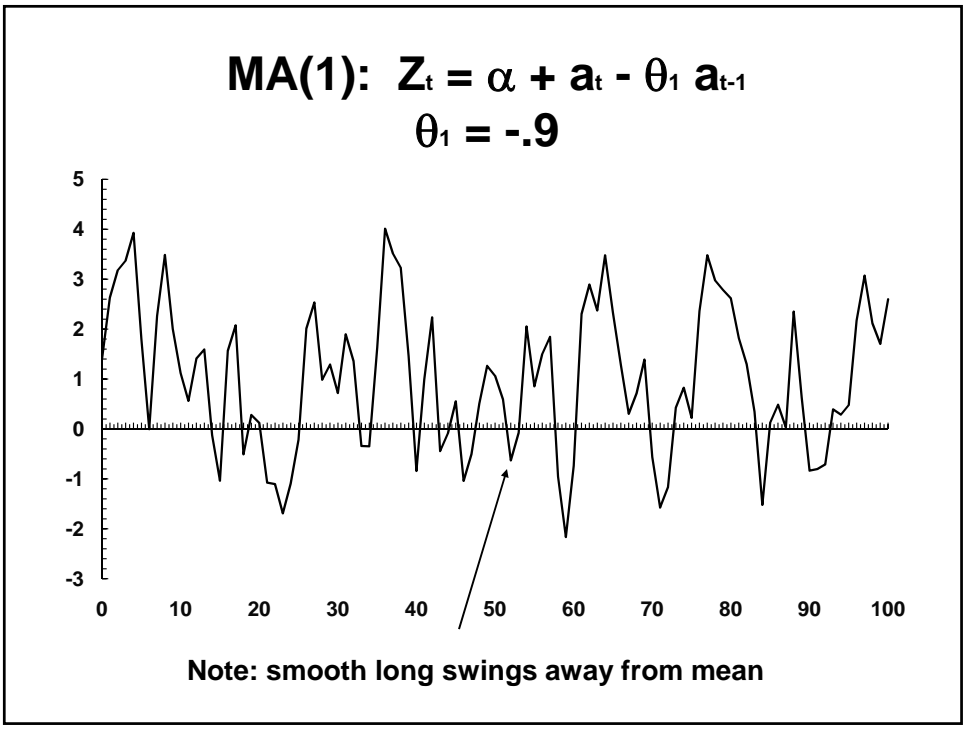
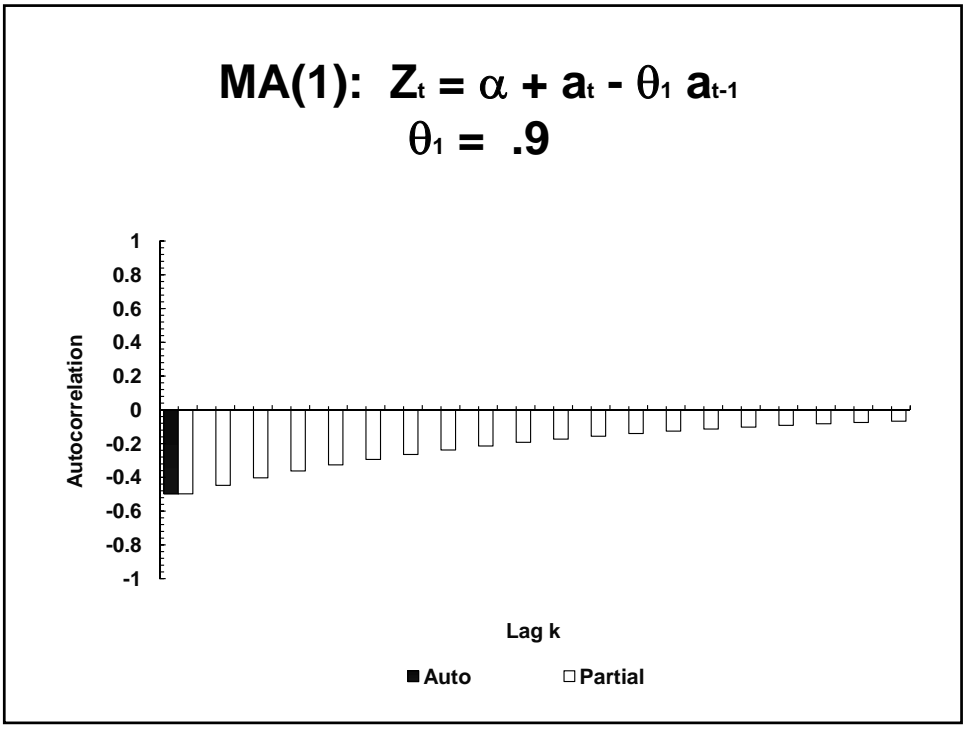
- 1) Autocorrelations decay or oscillate
- 2) Partial Autocorrelations cut-off after lag p, for AR(p) model
- 3) Stationarity is a big issue
 - very slow decay in autocorrelations
 - should you difference?

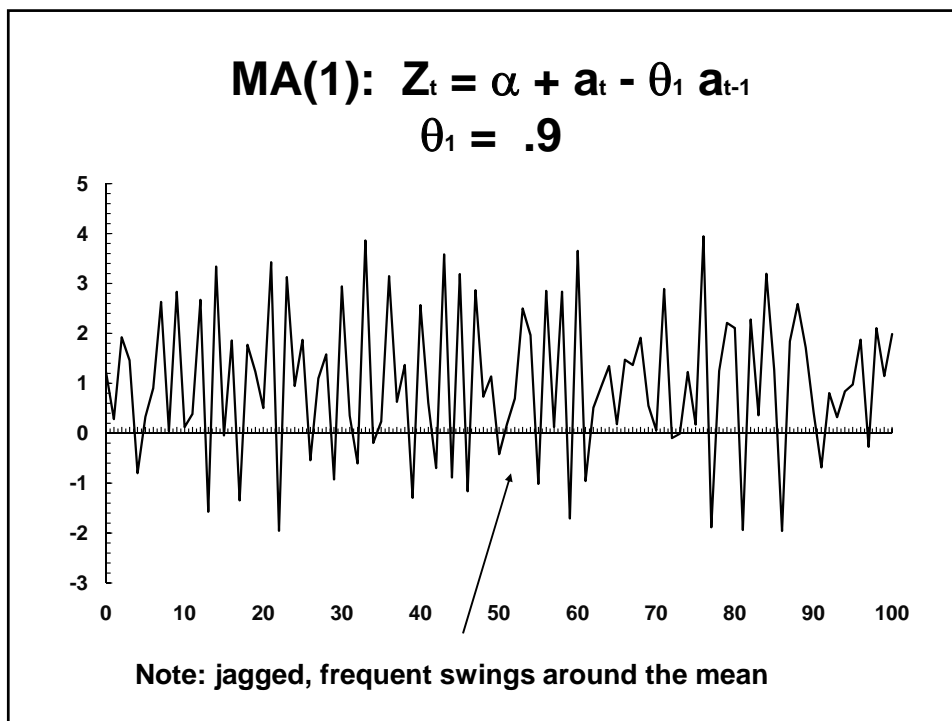
$$\text{MA}(1): Z_t = \alpha + a_t - \theta_1 a_{t-1}$$

$$\theta_1 = -.9$$









Moving Average Models: Summary

- 1) Autocorrelations cut off after lag q for MA(q) model
- 2) Partial autocorrelations decay or oscillate

Autoregressive Moving Average Models

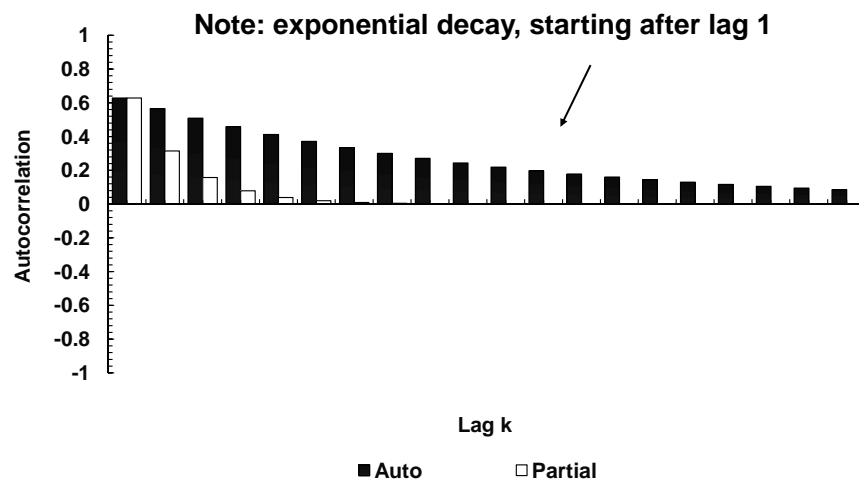
$$\text{ARMA}(p,q): Z_t = \alpha + \phi_1 Z_{t-1} + \dots + \phi_p Z_{t-p} + a_t - \theta_1 a_{t-1} - \dots - \theta_q a_{t-q}$$

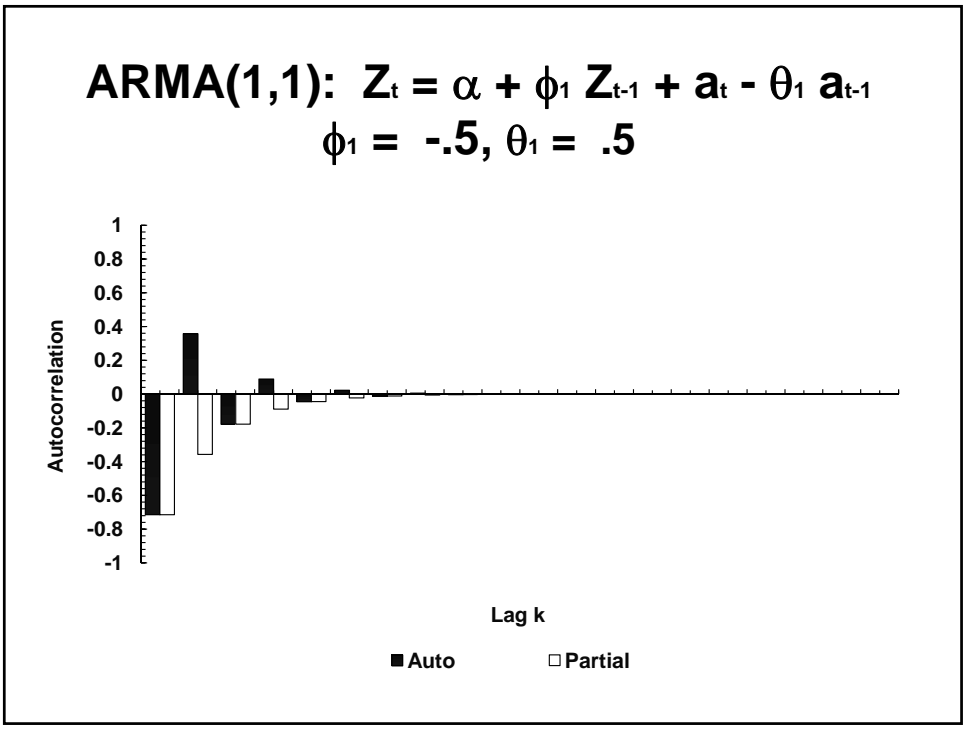
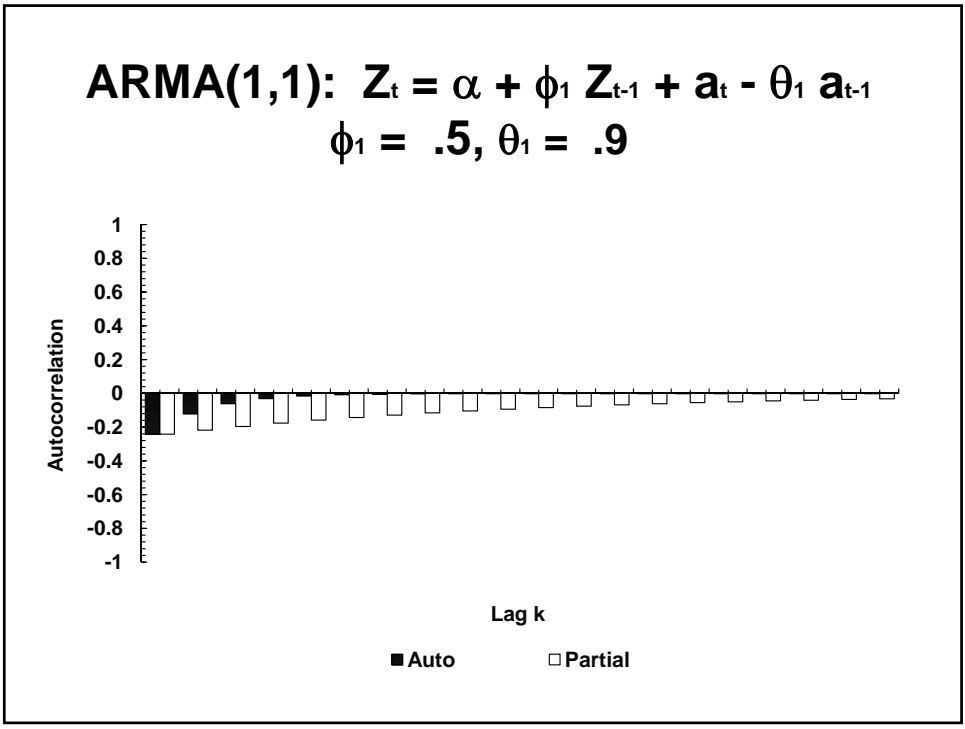
Combines both AR & MA characteristics:

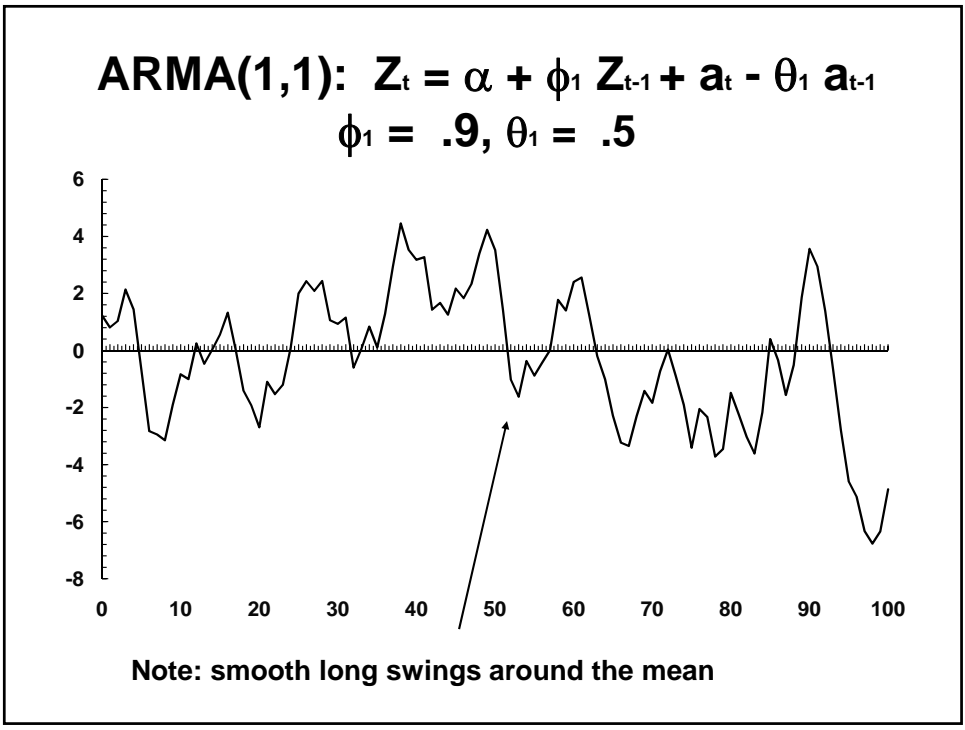
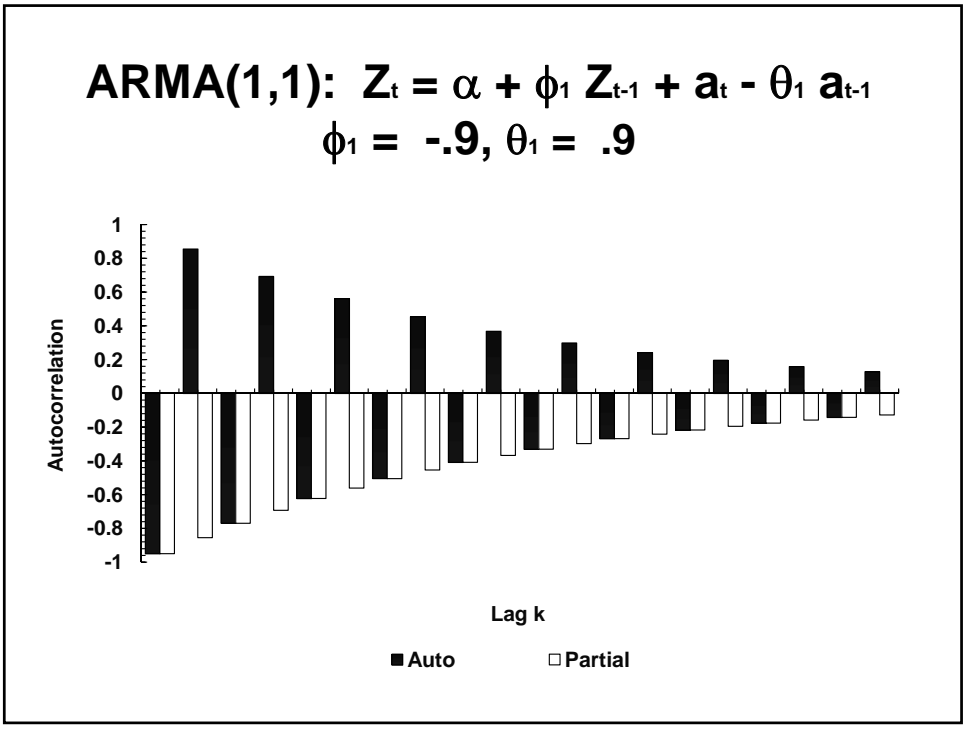
- equivalent to infinite order MA process
 - if stationary
- equivalent to infinite order AR process
 - if “invertible”

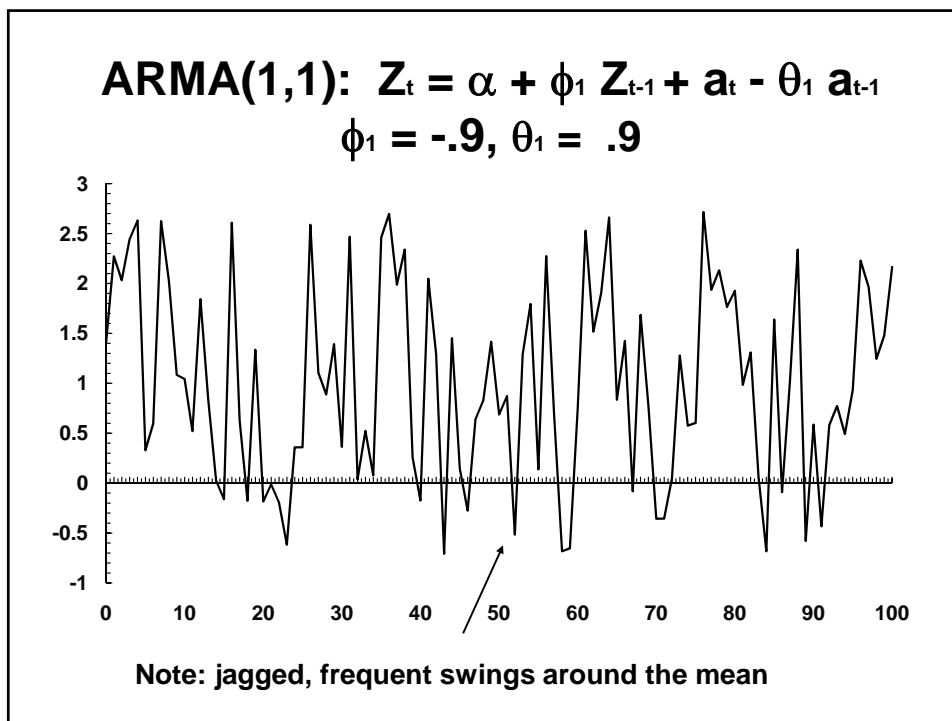
$$\text{ARMA}(1,1): Z_t = \alpha + \phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1}$$

$$\phi_1 = .9, \theta_1 = .5$$









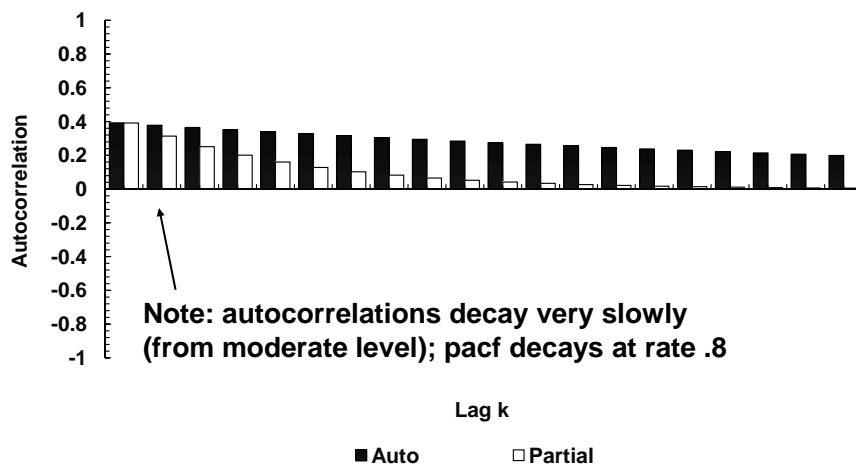
Autoregressive Moving Average Models: Summary

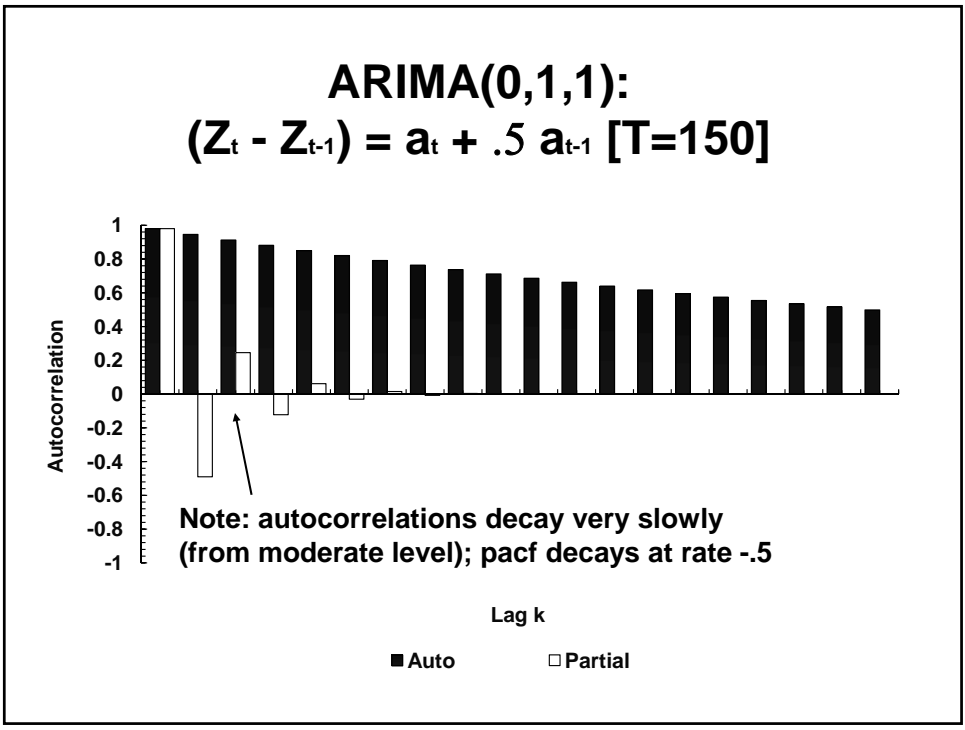
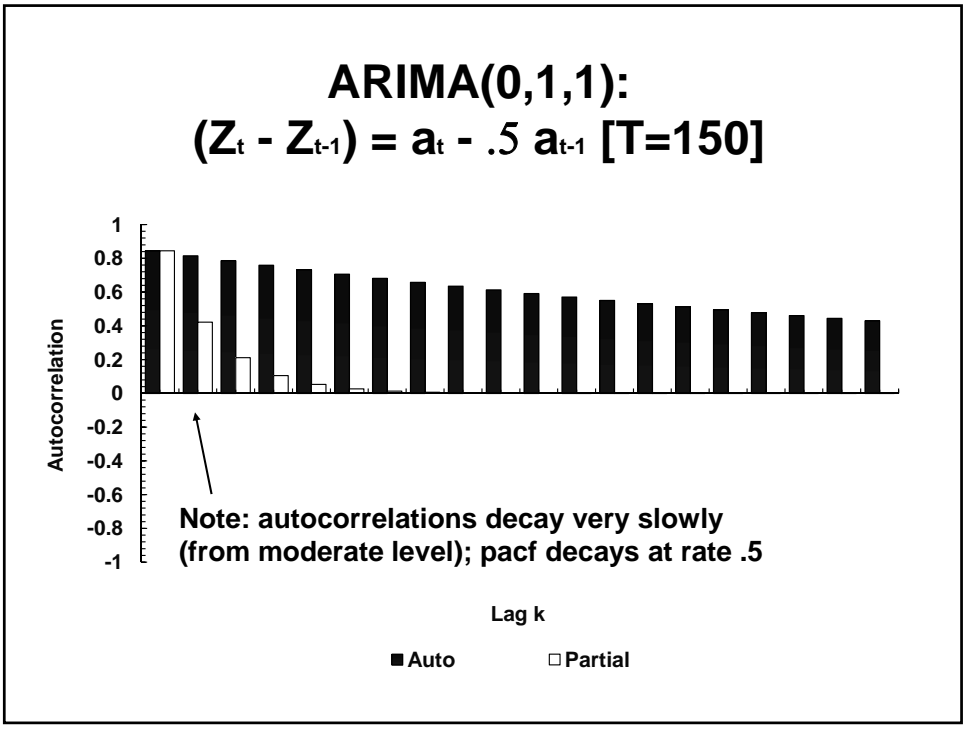
- 1) Autocorrelations decay or oscillate**
- 2) Partial Autocorrelations decay or oscillate**

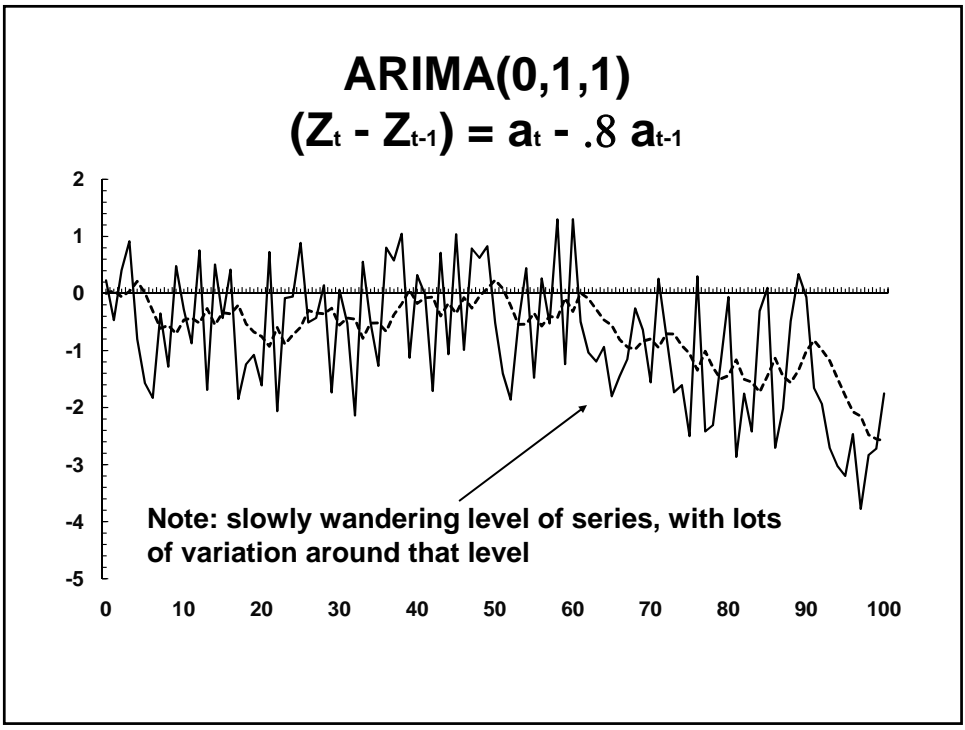
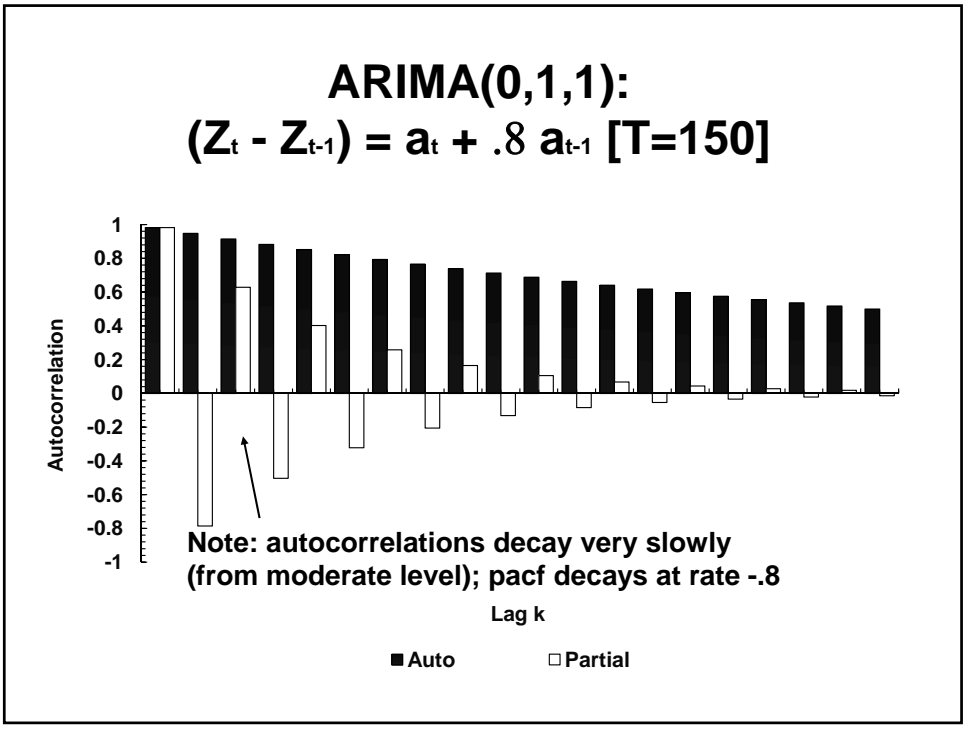
Autoregressive Integrated Moving Average ARIMA(p,d,q) Models

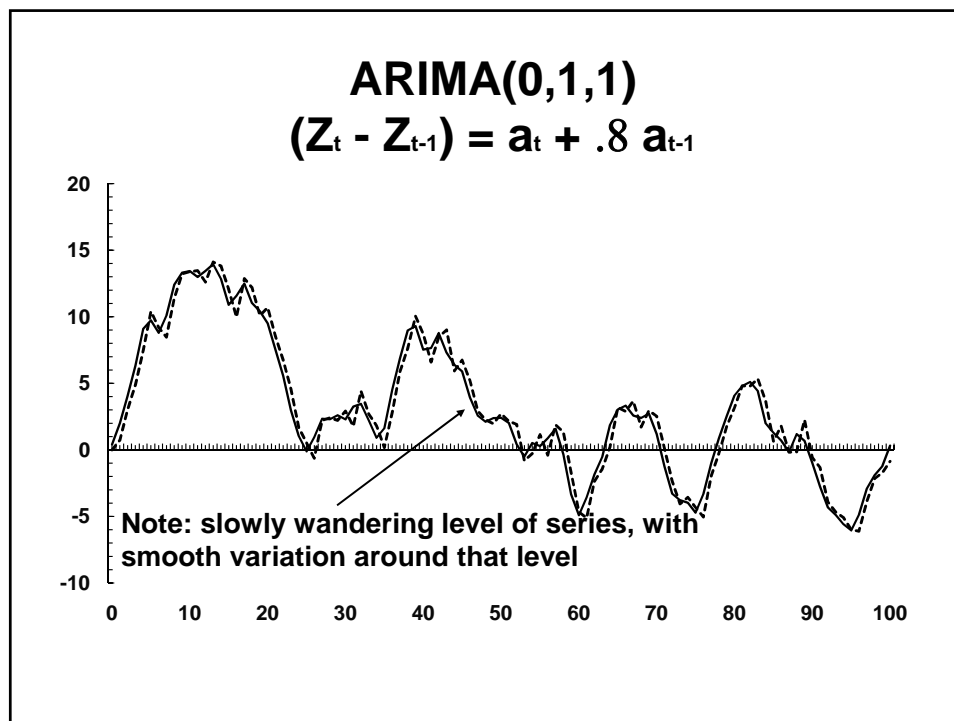
- 1) ARMA model in the d^{th} differences of the data
- 2) First step is to find the level of differencing necessary
- 3) Next steps are to find the appropriate ARMA model for the differenced data
- 4) Need to avoid “overdifferencing”

ARIMA(0,1,1):
 $(Z_t - Z_{t-1}) = a_t - .8 a_{t-1} [T=150]$









Integrated Moving Average Models: Summary

- 1) **Autocorrelations decay slowly**
 - initial level is determined by how close MA parameter is to one

- 2) **Partial Autocorrelations decay or oscillate**
 - determined by MA parameter

Links

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<http://schwert.simon.rochester.edu/A425/A425main.htm>