APS 425 – Fall 2015

GARCH Models

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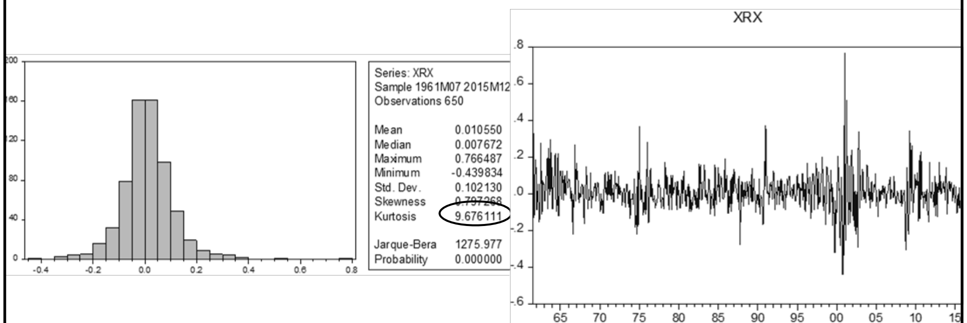
Autocorrelated Heteroskedasticity

- Suppose you have regression residuals
  - Mean = 0, not autocorrelated
- Then, look at autocorrelations of the absolute values of the residuals (or the squares of the residuals)
- This tells you if there is heteroskedasticity that varies over time

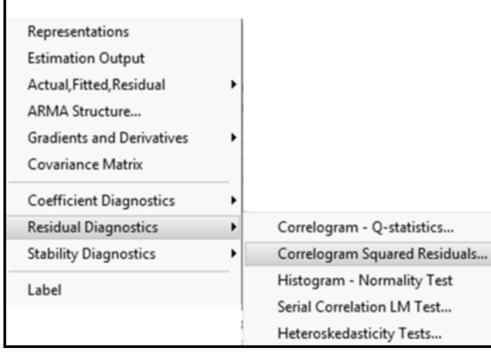
Example: Xerox Stock Returns

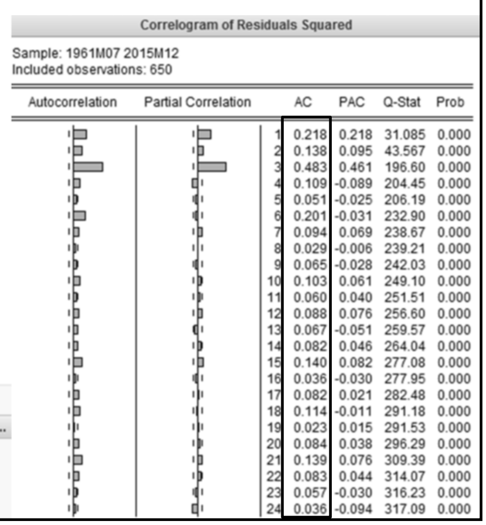
Kurtosis and wide, then narrow, bands in plot are hints of conditional heteroskedasticity



Example: Xerox Stock Returns

After estimating a regression with just a constant for the Xerox returns, the squared residuals have small positive autocorrelations





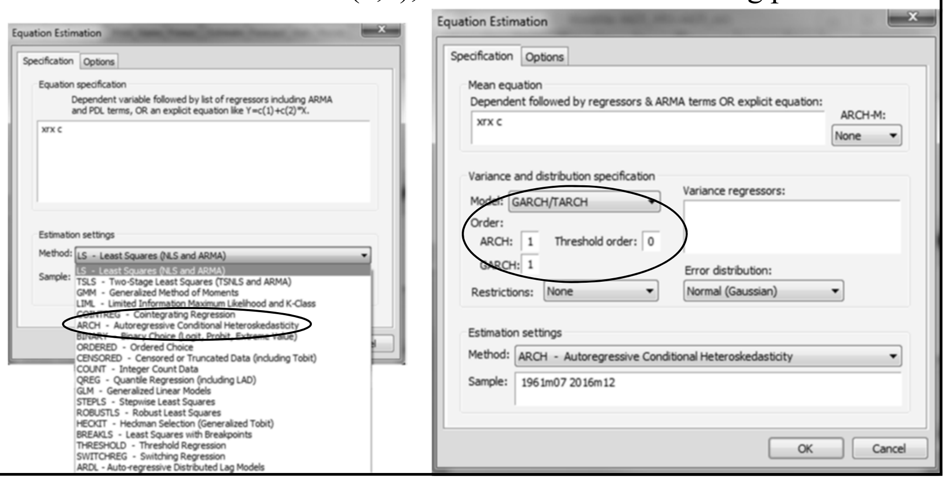
Correlogram of Residuals Squared

Sample: 1961M07 2015M12
Included observations: 650

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.218	0.218	31.085	0.000
		2	0.138	0.095	43.567	0.000
		3	0.483	0.461	196.60	0.000
		4	0.109	-0.089	204.45	0.000
		5	0.051	-0.025	206.19	0.000
		6	0.201	-0.031	232.90	0.000
		7	0.094	0.069	238.67	0.000
		8	0.029	-0.006	239.21	0.000
		9	0.065	-0.028	242.03	0.000
		10	0.103	0.061	249.10	0.000
		11	0.060	0.040	251.51	0.000
		12	0.088	0.076	256.60	0.000
		13	0.067	-0.051	259.57	0.000
		14	0.082	0.046	264.04	0.000
		15	0.140	0.082	277.08	0.000
		16	0.036	-0.030	277.95	0.000
		17	0.082	0.021	282.48	0.000
		18	0.114	-0.011	291.18	0.000
		19	0.023	0.015	291.53	0.000
		20	0.084	0.038	296.29	0.000
		21	0.139	0.076	309.39	0.000
		22	0.083	0.044	314.07	0.000
		23	0.057	-0.030	316.23	0.000
		24	0.036	-0.094	317.09	0.000

GARCH Model

Default model is GARCH(1,1), which is not a bad starting point



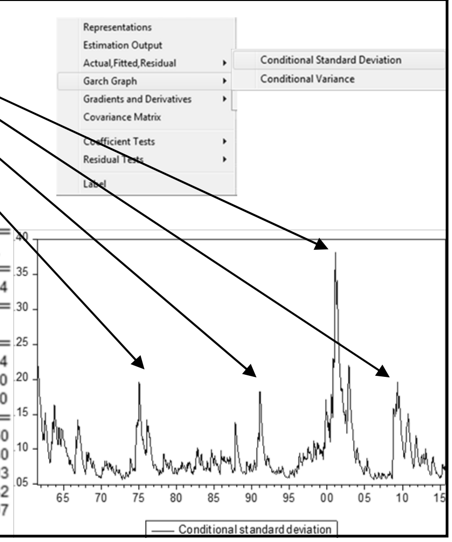
GARCH Model

Conditional sd graph shows brief periods of high volatility

Dependent Variable: XRX
Method: ML ARCH - Normal distribution (BHHH / EViews legacy)
Sample (adjusted): 1961M08 2015M09
Included observations: 650 after adjustments
Convergence achieved after 13 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.011202	0.003163	3.541832	0.0004
Variance Equation				
C	0.000600	0.000169	3.550892	0.0004
RESID(-1)^2	0.179105	0.029703	6.029955	0.0000
GARCH(-1)	0.756886	0.038570	19.62376	0.0000

R-squared	-0.000041	Mean dependent var	0.010550
Adjusted R-squared	-0.000041	S.D. dependent var	0.102130
S.E. of regression	0.102133	Akaike info criterion	-2.014293
Sum squared resid	6.769760	Schwarz criterion	-1.986742
Log likelihood	658.6451	Hannan-Quinn criter.	-2.003607
Durbin-Watson stat	1.960708		



GARCH(1,1) Model

$R_t = \mu + \varepsilon_t$

$\varepsilon_t \sim N(0, \sigma_t^2)$

$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$

Where
- μ is the mean of the returns
- σ_t^2 is the variance of the errors at time t
- ε_{t-1}^2 is the squared error at time t-1
- $\omega / (1 - \beta_1 - \alpha_1)$ is the unconditional variance
- α_1 is the first (lag 1) ARCH parameter
- β_1 is the first (lag 1) GARCH parameter

GARCH(1,1) Model

This looks a lot like an ARMA(1,1) model for the squared errors (as deviations from their forecasts), $\nu_t = (\varepsilon_t^2 - \sigma_t^2)$

$\varepsilon_t^2 = \omega + (\alpha_1 + \beta_1) \varepsilon_{t-1}^2 + \nu_t - \beta_1 \nu_{t-1}$

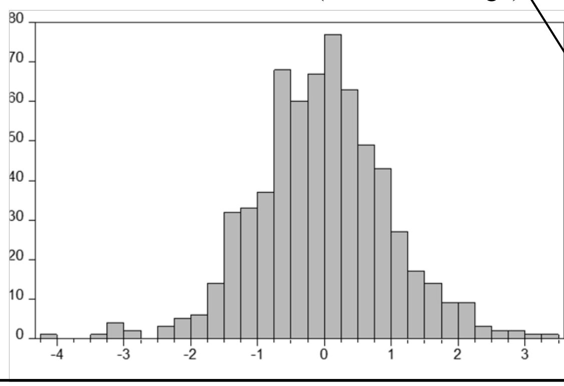
Often the GARCH parameter β_1 is close to 1, implying that the movements of the conditional variance away from its long-run mean last a long time

For Xerox $\beta_1 = .76$ and $\alpha_1 = .18$, so the implied AR(1) parameter is about .94 and the MA(1) coefficient is .76

GARCH Model Diagnostics

In Eviews, most of the residual diagnostics for GARCH models are in terms of the standardized residuals [which should be N(0,1)]

Note that kurtosis is smaller (still not 3, though)

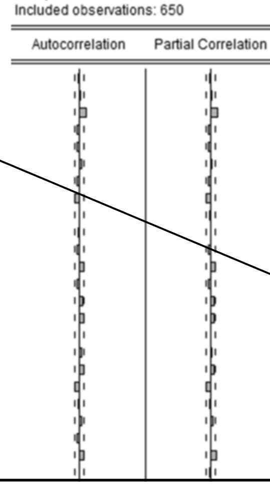


Series: Standardized Residuals
Sample 1961M08 2015M09
Observations 650

Mean	-0.036325
Median	-0.039475
Maximum	3.260330
Minimum	-4.198760
Std. Dev.	1.000238
Skewness	-0.070508
Kurtosis	3.982475
Jarque-Bera	26.68097
Probability	0.000002

GARCH Model Diagnostics

The correlogram for the standardized squared residuals now looks better



Correlogram of Standardized Residuals Squared

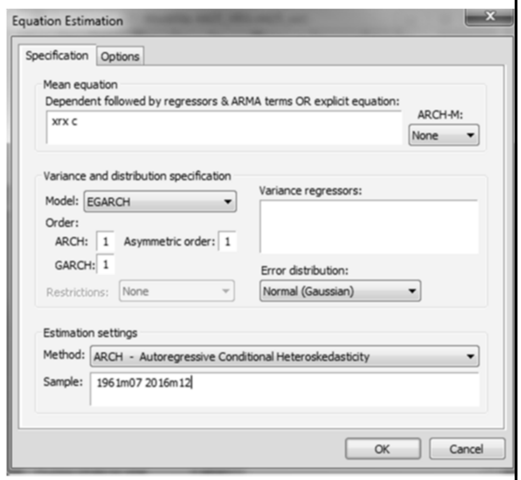
Sample: 1961M07 2015M12
Included observations: 650

	AC	PAC	Q-Stat	Prob*
1	-0.012	-0.012	0.0958	0.757
2	0.010	0.010	0.1645	0.921
3	0.098	0.099	6.5123	0.089
4	-0.028	-0.025	7.0098	0.135
5	-0.027	-0.030	7.4925	0.187
6	0.027	0.018	7.9849	0.239
7	-0.048	-0.042	9.5136	0.218
8	-0.069	-0.066	12.640	0.125
9	-0.008	-0.014	12.680	0.178
10	-0.012	-0.001	12.777	0.236
11	-0.038	-0.026	13.724	0.249
12	0.074	0.071	17.395	0.135
13	-0.046	-0.045	18.796	0.130
14	0.042	0.046	19.951	0.132
15	0.067	0.049	22.927	0.086
16	-0.004	0.001	22.939	0.115
17	0.028	0.019	23.44[illegible]	0.135
18	0.080	0.065	27.789	0.065
19	-0.073	-0.066	31.381	0.037
20	-0.017	-0.020	31.582	0.048
21	0.038	0.025	32.558	0.051
22	-0.032	-0.007	33.254	0.058
23	0.072	0.089	36.754	0.034
24	0.004	-0.010	36.768	0.046

EGARCH(1,1) Model

This model basically models the log of the variance (or standard deviation) as a function of the lagged log(variance/std dev) and the lagged absolute error from the regression model

It also allows the response to the lagged error to be asymmetric, so that positive regression residuals can have a different effect on variance than an equivalent negative residual



EGARCH(1,1) Model

"GARCH" is the variance the residuals at time t

The persistence parameter, c(5), is very large, implying that the variance moves slowly through time

The asymmetry coefficient, c(4), is negative, implying that the variance goes up more after negative residuals (stock returns) than after positive residuals (returns)

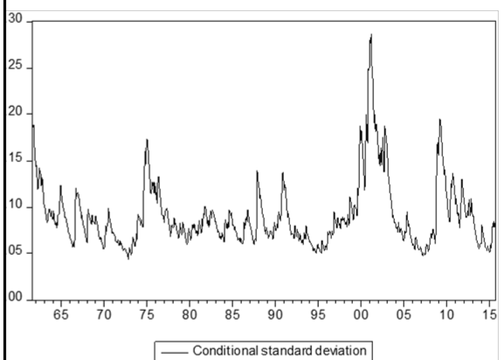
Dependent Variable: XRX
Method: ML ARCH - Normal distribution (BHHH / EViews legacy)
Sample (adjusted): 1961M08 2015M09
Included observations: 650 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.007122	0.003072	2.318598	0.0204
Variance Equation				
C(2)	-0.417248	0.094184	-4.430131	0.0000
C(3)	0.245062	0.043809	5.593887	0.0000
C(4)	-0.142665	0.021206	-6.727576	0.0000
C(5)	0.953778	0.014877	64.11023	0.0000

R-squared	-0.001129	Mean dependent var	0.010550
Adjusted R-squared	-0.001129	S.D. dependent var	0.102130
S.E. of regression	0.102188	Akaike info criterion	-2.061785
Sum squared resid	6.777124	Schwarz criterion	-2.027347
Log likelihood	675.0802	Hannan-Quinn criter.	-2.048428
Durbin-Watson stat	1.958577		

EGARCH Model Diagnostics

The correlogram for the standardized squared residuals still looks pretty good



Correlogram of Standardized Residuals Squared

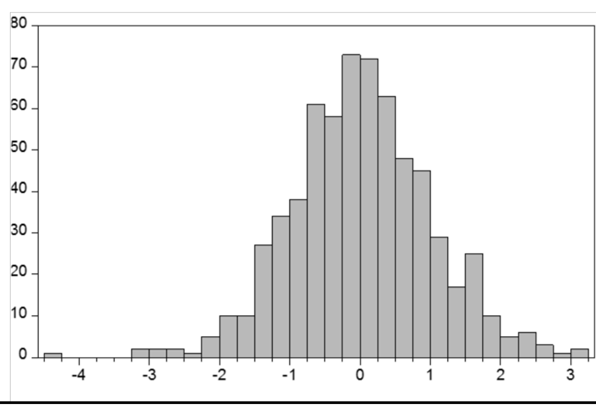
Sample: 1961M07 2015M12
Included observations: 650

	AC	PAC	Q-Stat	Prob*
1	-0.032	-0.032	0.6676	0.414
2	0.017	0.016	0.8538	0.653
3	0.081	0.082	5.1070	0.164
4	-0.039	-0.034	6.1115	0.191
5	-0.001	-0.006	6.1122	0.295
6	0.015	0.010	6.2669	0.394
7	-0.024	-0.017	6.6420	0.467
8	-0.074	-0.077	10.272	0.246
9	0.020	0.014	10.526	0.310
10	-0.010	-0.002	10.593	0.390
11	-0.007	0.002	10.626	0.475
12	0.054	0.046	12.543	0.403
13	-0.044	-0.039	13.831	0.386
14	0.036	0.034	14.709	0.398
15	0.043	0.036	15.968	0.384
16	-0.004	0.002	15.977	0.455
17	0.029	0.021	16.532	0.487
18	0.095	0.092	22.554	0.208
19	-0.064	-0.055	25.277	0.152
20	-0.012	-0.019	25.377	0.187
21	0.042	0.025	26.550	0.186
22	-0.017	0.009	26.745	0.221
23	0.092	0.096	32.520	0.090
24	0.008	0.004	32.568	0.114

EGARCH Model Diagnostics

In Eviews, most of the residual diagnostics for GARCH models are in terms of the standardized residuals [which should be N(0,1)]

Note that kurtosis is smaller (still not 3, though)

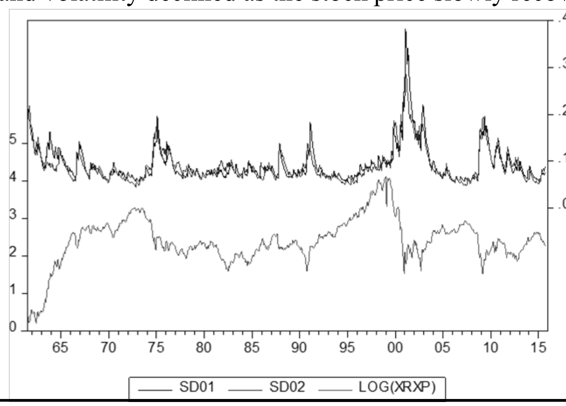


Series: Standardized Residuals
Sample 1961M08 2015M09
Observations 650

Mean	0.018241
Median	0.007184
Maximum	3.212804
Minimum	-4.453100
Std. Dev.	1.000641
Skewness	-0.053954
Kurtosis	3.804555
Jarque-Bera	17.84665
Probability	0.000133

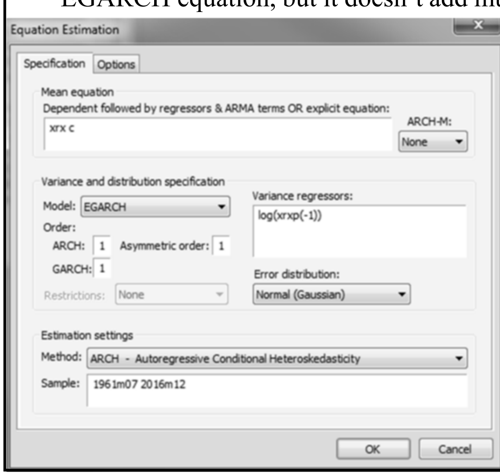
EGARCH Model Extensions

Plotting the log of Xerox's stock price on the right axis, versus the two estimates of the conditional standard deviation [from GARCH(1,1) and EGARCH(1,1)], you can see that the crash in the stock price occurs at the same time as the spike in volatility, and volatility declined as the stock price slowly recovered



EGARCH Model Extensions

Include the lagged log of Xerox's stock price as an additional variable in the EGARCH equation, but it doesn't add much



Dependent Variable: XRX
Method: ML ARCH - Normal distribution (BHHH / EViews legacy)
Sample (adjusted): 1961M08 2015M09
Included observations: 650 after adjustments
Convergence achieved after 16 iterations
Presample variance: backcast (parameter = 0.7)
LOG(GARCH) = C(2) + C(3)*ABS(RESID(-1)/@SQRT(GARCH(-1))) + C(4)
*RESID(-1)/@SQRT(GARCH(-1)) + C(5)*LOG(GARCH(-1)) + C(6)
*LOG(XRXP(-1))

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.006728	0.003161	2.128250	0.0333
Variance Equation				
C(2)	-0.418570	0.096440	-4.340201	0.0000
C(3)	0.244510	0.045465	5.378024	0.0000
C(4)	-0.152176	0.025335	-6.006611	0.0000
C(5)	0.947445	0.017511	54.10512	0.0000
C(6)	-0.011958	0.012929	-0.924906	0.3550

R-squared	-0.001403	Mean dependent var	0.010550
Adjusted R-squared	-0.001403	S.D. dependent var	0.102130
S.E. of regression	0.102202	Akaike info criterion	-2.059703
Sum squared resid	6.778981	Schwarz criterion	-2.018377
Log likelihood	675.4034	Hannan-Quinn criter.	-2.043673
Durbin-Watson stat	1.958041		

Links

Xerox Stock Return GARCH dataset:
http://schwert.ssb.rochester.edu/A425/A425_xrx.wf1

Return to APS 425 Home Page:
http://schwert.ssb.rochester.edu/A425/A425main.htm