

Bond Pricing and Interest Rates

- Government bond prices represent the time value of money
 - In the US, pictures of George Washington
 - No (or very small) default risk
 - Simple application of NPV analysis
 - Nominal interest rates
 - Real interest rates + expected inflation

Coupon Bonds & “Zeroes”

- Zero coupon bonds are pure discount securities
 - All US Treasury bills (with maturities at issue less than 1 year) are pure discount bonds
 - The only payoff is the \$10,000 (or multiples) paid at the maturity date
$$P = \$10,000 / (1 + r)^k$$
 - P = price of bill
 - r = effective annual interest rate
 - k = number of years to maturity
 - (e.g., $k = 1/2$ for a six month bill)

Coupon Bonds & “Zeroes”

- Example:

- Six month Tbill selling at \$9,900

$$9,900 = \$10,000 / (1 + r)^{1/2}$$

$$1 + r = [10,000/9,900]^2$$

So $r = 0.0203$ (or 2.03% effective annual rate)

Coupon Bonds & “Zeroes”

- Coupon bonds pay cash (coupon) payments every six months until maturity, plus the principal repayment at maturity
 - You can think of it as a portfolio of zero coupon bonds maturing every six months to maturity (with equal face value) plus a larger face value discount bond with a payoff at the maturity date
 - The weighted average maturity is shorter than for a zero coupon bond with the same maturity date
 - This is called Macaulay’s duration measure

Coupon Bonds & “Zeroes”

- Example:

- Suppose you had a Treasury note maturing in March 2018 with a coupon rate of 2.875% per year

- Current price (average of bid and ask prices from 4/24/2017 WSJ) is 101.6563

- You also have Treasury bills maturing in September 2017 and March 2018 with yields of:

- 9/28/2017: $r = 0.873$
 - Price = $100/(1+r)^{[157/365]} = 99.62682$
- 3/29/2018: $r = 1.003$
 - Price = $100/(1+r)^{[339/365]} = 99.07737$

Coupon Bonds & “Zeroes”

- Present value of coupons:

- 9/28/2017: $\$1.432 = .9962682 * \$2.875/2$ per \$100 face value
- 3/29/2018: $\$1.424 = .9907737 * \$2.875/2$ per \$100 face value

- Present value of principal:

- 3/29/2018: $\$99.077 = .9907737 * \100 per \$100 face value

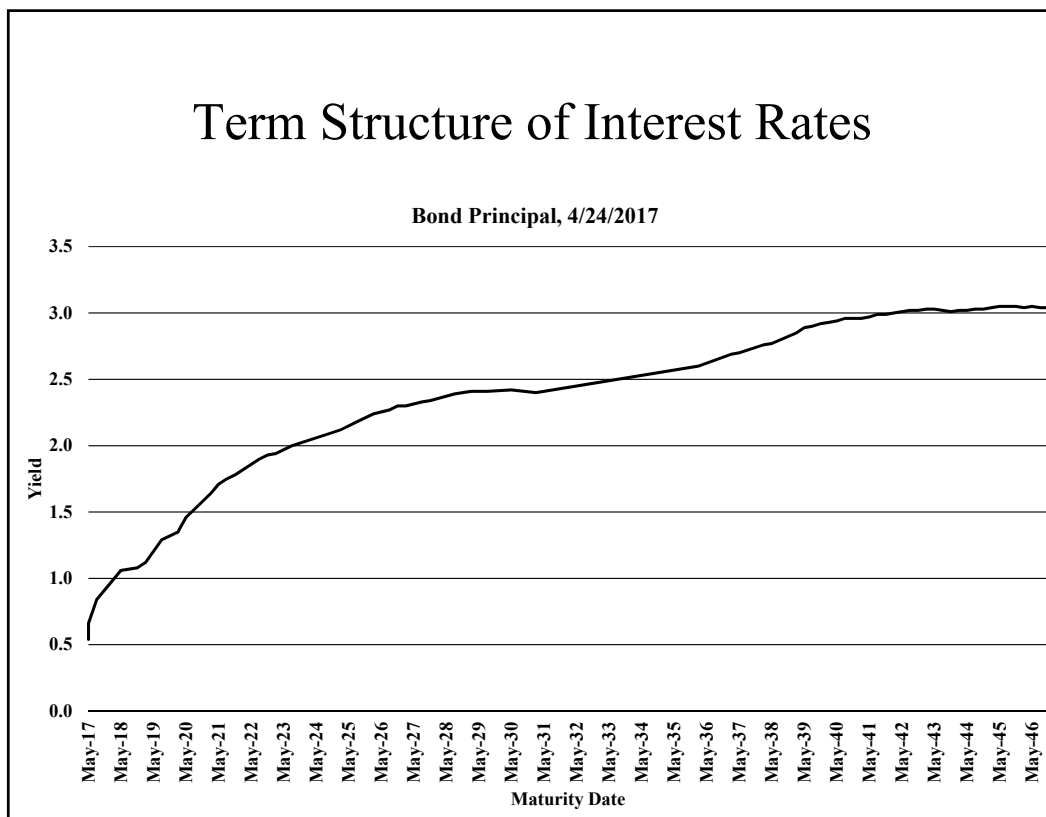
- Total value of coupon bond:

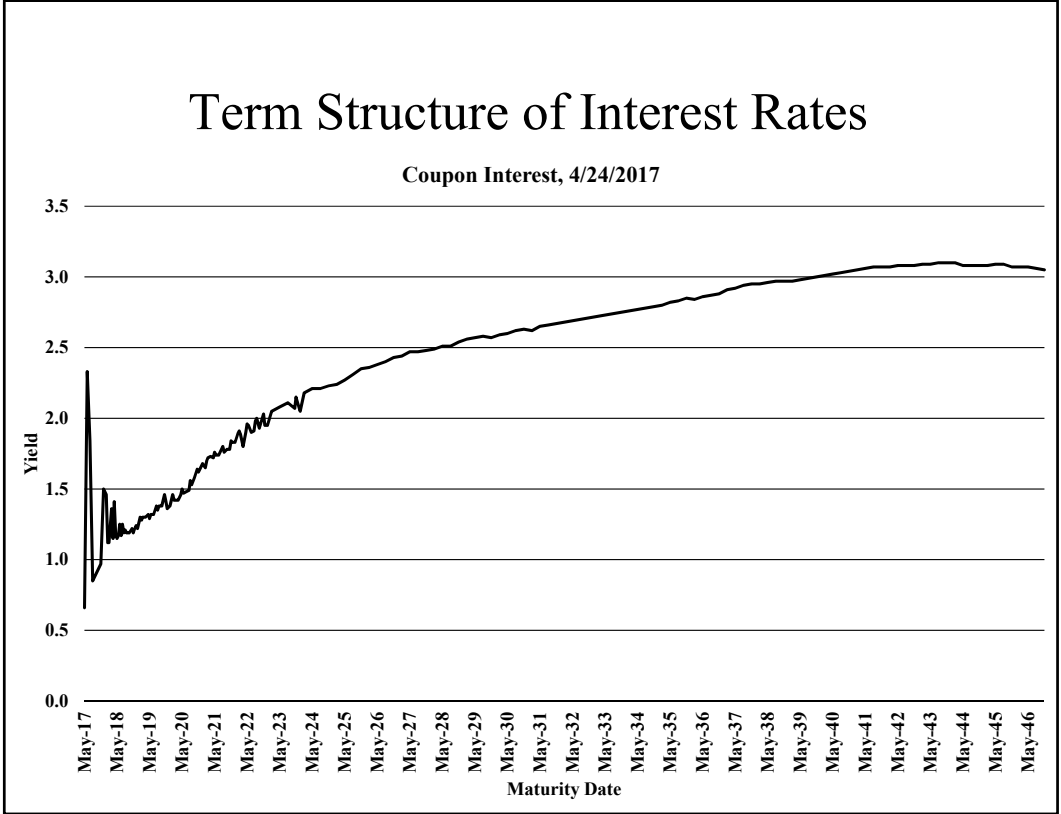
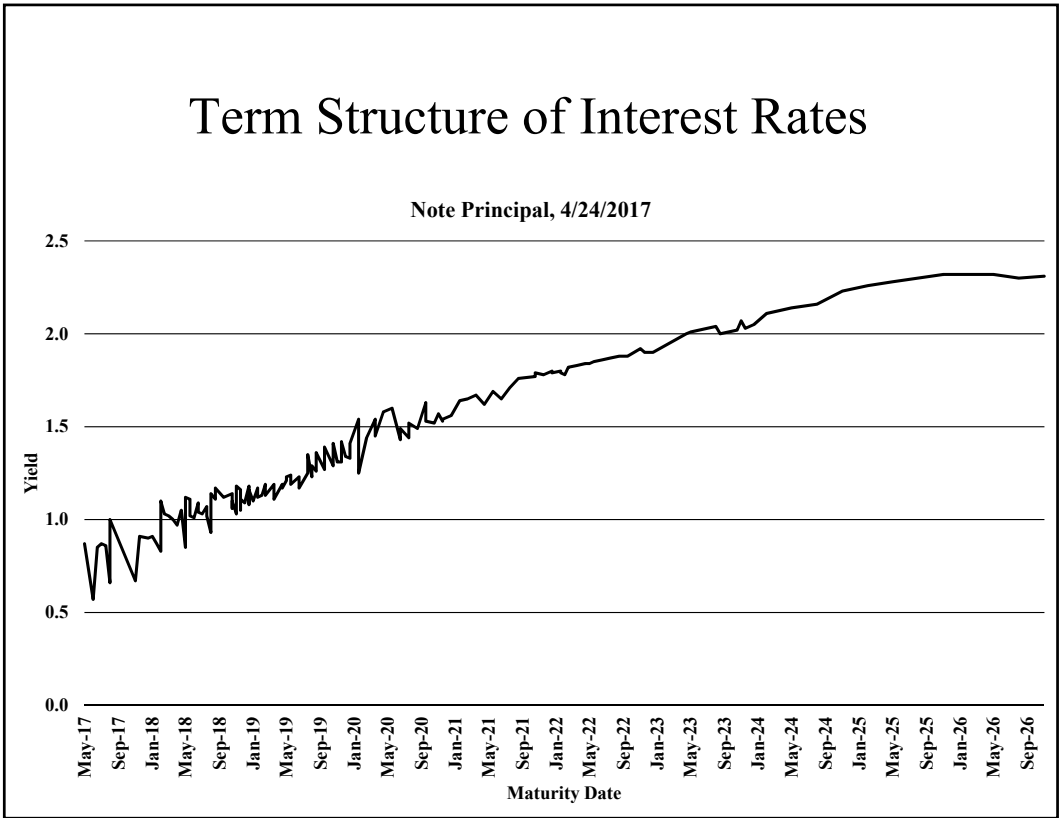
- 3/29/2018: $\$101.934 = \$99.077 + 1.432 + 1.424$
per \$100 face value
- Compared to ask price of \$101.6641
 - Note that the quoted yield on the note is 1.075%, which is a weighted average of the yields on the coupon and principal cash flows

Term Structure of Interest Rates

- Term structure refers to the shape of the plot of yields to maturity as a function of the maturity date
 - Discount term structure reflects yields on Tbills and zero coupon (strip) bonds
 - This is the price of receiving \$1 at some time in the future
 - Coupon term structure reflects yields on coupon bonds
 - These are each weighted averages of the yields associated with the intermediate and final cash flows (i.e., pretty meaningless)

Term Structure of Interest Rates





Term Structure and Forward Rates

- Think of the yield on a 2-period bond in terms of the yield on the 1-period bond and the forward rate of interest from period 1 to period 2

$$(1 + y_2)^2 = (1 + y_1)(1 + r_2)$$

where y_1 is the yield to maturity on a 1-period strip and r_2 is the forward rate for period 2

Term Structure and Forward Rates

- In general:

$$(1 + y_k)^k = (1 + y_{k-1})^{k-1} (1 + r_k)$$

where y_k is the yield to maturity on a k-period strip and r_k is the forward rate for period k (note: $r_1 = y_1$)

$$(1 + y_k)^k = (1 + r_1)(1 + r_2)(1 + r_3) \dots (1 + r_k)$$

So the yield to maturity is simply a geometric average of the forward rates

Term Structure and Forward Rates

- If the term structure is rising, then the forward rate is above the spot rate

$$r_k > y_k$$

and vice versa

- Holding period returns from period to period depend on movements of the term structure
 - Increases in interest rates cause bond prices to fall
 - Shift in the term structure
 - Movements along the term structure (no shifts) cause returns to equal forward rates

Term Structure and Forward Rates

- Note that forward rates are not just a theoretical concept
- You can invest in a forward contract, buying a k-period strip and short selling a (k-1) period strip
 - You receive the interest rate r during period k
- We will see this concept again when we get to a discussion of futures contracts

Interest Rates and Inflation

- One reason why nominal interest rates are different for different maturities (at the same point in time), and why nominal interest rates of the same maturity are different at different points in time is

EXPECTED INFLATION

real rates = nominal rates – expected inflation

Term Structure and Profit Opportunities

- Term structure may be the best example of an efficient market
 - Lots of traders
 - Information is widely available and no one has cheap access to better information about the macroeconomy
 - Transaction costs are very low
 - Scope of uncertainty is limited
 - Set of possible outcomes is small
 - Unlike future events that affect the value of corporate securities
- This implies that “profit opportunities” should be scarce

Term Structure and Profit Opportunities

- Means that it should not pay to “play” the term structure
 - buy long-term bonds because they have higher yields?
 - **Why Not?**
 - If rates rise in the future, bond prices will fall, so returns will be lower than implied by forward rates
 - Also, rising forward rates may imply that inflation or real interest rates will be much higher in the future than they are today

Term Structure and Profit Opportunities

- **Humorous example (sort of):**
 - Original Bill Clinton Budget Plan:
 - Largest single source of “budget cuts” was from borrowing at the short end of the term structure!
 - Is this an “expenditure cut”?
 - In the short-run, less cash flows leave the Treasury
 - In the longer run, if the upward sloping term structure correctly implies that short-term rates will rise, the cost of short-term borrowing will rise
 - Future administrations would have to pay for the decision to borrow short today
 - No free lunch (just put it on someone else’s bill . . .)

Corporate Bonds and Default Risk

- Rating agencies (Moody's S&P, Fitch, etc.) publish ratings for corporate, municipal and structured finance debt
 - Intended to reflect default risk (and losses given default)
 - Aaa is highest Moody's rating
 - Baa is lowest "investment grade" bond
 - Eligible for purchase by most banks, insurance companies, etc.
 - Below Baa are "junk" (high yield) bonds
 - Somewhat like equity since their payoffs are strongly linked to the health of the issuer
 - Yields are higher (and prices lower) to reflect more default risk

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