Futures Pricing

Futures contract is an agreement to buy
  – a fixed amount (& quality) of a product
  – at a specified price
  – at a specified time in the future

At the time the contract is begun, no money changes hands (no investment)
  – the value of the contract is returned to zero
  – every day by "settling up"
    • transferring money from the seller(buyer) to the buyer(seller) if the futures price goes up(down)

Forward Pricing

Forward contract is an agreement to buy
  – a fixed amount (& quality) of a product
  – at a specified price
  – at a specified time in the future
  – No daily settling up

Thus, forwards and futures differ only because of the daily settling up provision
  – Important to reduce default risk
  – Randomness in short-term interest rates causes differences in forward and future values
Financial Futures Contracts:
Stocks

Futures contract on the S&P 500 index:

- Chicago Mercantile Exchange (MERC)
- $50 times the index value (E-mini)
- very actively traded
- used for hedging large stock portfolios
  - portfolio insurance
- settled in cash

S&P Index Futures:
Arbitrage Pricing

Alternative strategies:

(1) buy S&P futures at a price $F_0$ & Treasury bills with an interest rate of $r_f$ equivalent to buying the stock $S_0$
  - if the price of the index at maturity is $S_T$, then the payoff to (1) is:
    $$[S_T - F_0] + [(1 + r_f) S_0]$$

(2) buy S&P index at a price $S_0$ and receive dividend $D$
  - the payoff to strategy (2) is:
    $$[S_T + D]$$
S&P Index Futures: Arbitrage Pricing

Alternative strategies:

Equating the payoffs to these two strategies gives:

\[ F_0 = (1 + r_f) S_0 - D \]

In pseudo-return form:

\[ \frac{F_0 - S_0}{S_0} = r_f - \frac{D}{S_0} \]

The percent basis equals the difference between the interest rate \( r_f \) and the dividend yield \( d = \frac{D}{S_0} \) on the index

If the interest rate is less than the dividend yield, the futures price should be lower than the spot price
S&P Index Futures: 
Arbitrage Pricing

<table>
<thead>
<tr>
<th>3 month maturity</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_0 )</td>
<td>2721.33</td>
</tr>
<tr>
<td>( F_0 )</td>
<td>2722.80</td>
</tr>
<tr>
<td>( r_f ) (annual rate Tbill)</td>
<td>2.262</td>
</tr>
<tr>
<td>( r_f ) (annual rate Eurodollar)</td>
<td>2.731</td>
</tr>
<tr>
<td>annual S&amp;P dividend yield, ( d )</td>
<td>1.840</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>implied ( d ) (Tbill)</td>
<td>2.260</td>
</tr>
<tr>
<td>implied ( d ) (Eurodollar)</td>
<td>2.729</td>
</tr>
<tr>
<td>implied ( r_f ) (annual rate)</td>
<td>1.838</td>
</tr>
</tbody>
</table>

So there are two ways to evaluate the formula:

Assume the interest rate you are using is “correct” and solve for the implied dividend yield, \( d \)

Or, assume the dividend yield is correct and solve for the implied interest rate, \( r_f \)

The basis is consistent with a higher “interest rate” (e.g., LIBOR), or with lower dividend yield, \( d \)

Financial Futures Contracts: 
Bonds

Futures contract on Treasury Bonds:
– Chicago Mercantile Exchange (CME)
– $100,000 face value
– Maturity of 15 to 25 years

Futures contract on Eurodollars:
– Chicago Mercantile Exchange (CME/IMM)
– $1,000,000 principal value
– Maturity of 3 months
– 100 points minus 3 month LIBOR offered rate
  • LIBOR = 2.33\%, then price = 97.67
  • One interest rate basis point = $25
– Settled in cash
Financial Futures Contracts: Relation to Option Pricing

Buy Call Payoff

Sell Put Payoff

Financial Futures Contracts: Relation to Option Pricing

Buy Call & Sell Put Payoff

Buy Futures Payoff

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Financial Futures Contracts:  
Relation to Option Pricing

Long futures positions is equivalent to buying a call and selling a put with exercise prices equal to the futures price
  - selling the futures is equivalent to selling a call & buying a put

Payoff from futures contract is symmetric
  - you can make a lot of money if you guess right
  - but you can lose a lot of money if you are wrong

Other Futures Contracts

Financial:
  • other interest rates (Treasury notes, Fed Funds, etc.)
  • other US stock indexes (Nasdaq, DJIA, Russell)
  • US sector stock indexes (Energy, Tech, Health Care, etc.)
  • foreign stock indexes (Nikkei, China, FTSE, Brazil, India)
  • Foreign exchange (UK, Japan, Euro, etc.)
  • Metals (gold, silver, etc.)
  • Options on Futures
  • Options and Futures on Volatility (VIX)

Agricultural:
  • corn, wheat, soybeans, hogs, cattle, etc.

Others:
  • Weather, real estate
Options on Futures Contracts

Most Popular:
• Eurodollar options
• Treasury Note options (5 and 10-year maturity)
• Crude Oil
• Natural Gas (Europe)
• S&P 500 E-mini
• Corn
• Euro FX
• Gold
• VIX

Caveat Emptor

Financial futures are a cheap way to take on a lot of risk
• low transactions costs
• large leverage ( margins of 5 to 15%, versus 50% for stock purchases)
  • i.e., you have to provide about $100,000 of Treasury bills as collateral to bet $1,000,000 on stock or bond price moves

Unless you have (macroeconomic) "inside information," you should use futures (& options) for risk management
• hedging
Hedging with Financial Futures: Portfolio Insurance

Used to reduce the (market) risk of a large, well-diversified stock portfolio

1. buy put options on S&P index
   • puts lower bound on losses

2. sell call options on S&P index
   • increases value if stock prices fall or stay level, but you lose if stock prices rise a lot

3. sell S&P 500 futures contracts against a portion of your portfolio
   • as if you were investing in Tbills with that portion of your investment

Hedging with Agricultural Futures

Suppose you are a farmer in North Dakota who will have 100,000 bushels of northern spring wheat to sell in August

You want to hedge the price risk associated with this sale in the futures market

Do you take a long or short position in the futures market?

• Because your underlying exposure is a long wheat position, you offset that risk with a corresponding short position in the futures market
Hedging with Agricultural Futures

How many bushels of wheat do you want to short in the futures market?

The optimal size of your position (and the effectiveness of your hedge) will depend on the basis risk in the wheat futures market.

Basis Risk

basis = spot price of asset to be hedged - futures price of contract used to hedge

The basis can be different from zero for many reasons:
• underlying asset
• delivery date
• delivery location
Basis Risk

Let

\[ S_1 = \text{spot price at time } t_1 \]
\[ S_2 = \text{spot price at time } t_2 \]
\[ F_1 = \text{futures price at time } t_1 \]
\[ F_2 = \text{futures price at time } t_2 \]
\[ b_1 = \text{basis at time } t_1 \]
\[ b_2 = \text{basis at time } t_2 \]

Consider a hedger who knows an asset will be sold at time \( t_2 \) and opens a short hedge at time \( t_1 \). The net price obtained at time \( t_2 \) is

\[ S_2 + F_1 - F_2 = F_1 + (S_2 - F_2) \]
\[ = F_1 + b_2 \]

The value of \( F_1 \) is known at time \( t_1 \). Thus the only remaining risk is the risk associated with \( b_2 \).

This is called basis risk
Hedging with Basis Risk

If the basis is constant, then the optimal hedge amount is equal to your underlying exposure

\[ h = -Q = \frac{-100,000 \text{ bu.}}{(5000 \text{ bu per contract})} = -20 \text{ contracts} \]

• But, futures contracts in Kansas City are for delivery of hard red winter wheat in Kansas City, and the current delivery months are July and September

• Thus, the futures price can vary significantly from the spot price

Hedging with Basis Risk

You can often estimate basis risk with a linear regression

\[ \Delta P_s = \alpha + \beta \left( \Delta P_f \right) + e \]

The variance of \( e \) is a measure of the basis risk in your hedge portfolio

The optimal hedge amount \( h = -Q \beta \)

Suppose \( \beta = .93 \). Then,

\[ h = -Q \beta = \frac{-100,000(.93)}{5000} = -18.6 \text{ contracts} \]
Hedging with Basis Risk: Additional Issues

Which contract?

How many different contracts?

Use multiple regression

\[ \Delta P_s = \alpha + \beta_1 (\Delta P_{f1}) + \beta_2 (\Delta P_{f2}) + e \]

\[ h_1 = -Q \beta_1 \]

\[ h_2 = -Q \beta_2 \]

What futures delivery date should you choose?

Financial Futures: Questions

(1) If you had inside information about a specific company, how might you use options or futures to augment your investment strategy? Discuss:
   - options on individual stocks
   - options on market indexes
   - futures on market indexes

(2) If you wanted to adjust the risk of your company's pension fund portfolio, would you use options & futures to do this? Why, or why not?
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