Portfolio Theory and Practice

- Perhaps the most important concept from modern finance:

DIVERSIFICATION

- Don't put all your eggs in one basket . . .
- Basically driven by the benefits of spreading your money around among a variety of assets whose returns are not perfectly correlated

Algebra of Portfolio Returns

The return to a portfolio R(pt) is just a weighted average of the returns to the N individual assets in the portfolio:

$R(pt) = \Sigma\ w(it)\ R(it)$, (sum for $i = 1, \ldots N$), where

$R(it) = (P(it) + D(it) - P(it-1)) / P(it-1)$

and the portfolio weights w(it) sum to 1

$\Sigma\ w(it) = 1$

w(it) = the fraction of the value of the portfolio represented by the investment in asset i at time t-1

Expected Portfolio Returns

The expected return to a portfolio R(pt) is just a weighted average of the expected returns to the individual assets in the portfolio:

$$E[R(pt)] = \Sigma\ w(it)\ E[R(it)]$$

The variance of the return to a portfolio R(pt) is the doubly weighted average of all the pairwise covariances of the individual assets in the portfolio:

$$Var[R(pt)] = \Sigma\ \Sigma\ w(it)\ w(jt)\ cov[R(it), R(jt)]$$

for i, j = 1, … N

Variance of Portfolio Returns

Since $R(pt) = \Sigma\ w(it)\ R(it)$, the portfolio variance can be written as:

$$Var[R(pt)] = \Sigma\ w(it)\ cov[R(it), R(pt)]$$

for i = 1, … N. Dividing both sides of this equation by Var[R(pt)] gives the expression for the relative risk of asset i in portfolio p, sometimes called the "beta" coefficient:

$$1 = \Sigma\ w(it)\ cov[R(it), R(pt)] / Var[R(pt)]$$

So $\beta(i,p) = cov[R(it), R(pt)] / Var[R(pt)]$ averages to one across securities i in portfolio p

Investment Opportunity Set: 2 assets

- Suppose you have two assets, a and b, with the different expected returns, but the same standard deviations:

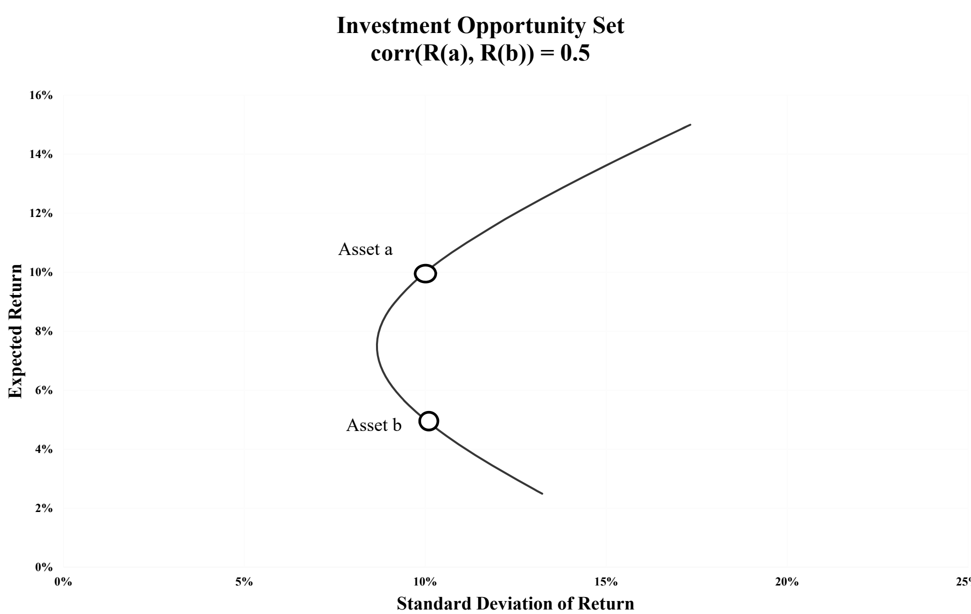
E[R(a)] = .10, E[R(b)] = .05

Var[R(a)] = Var[R(b)] = .01, so

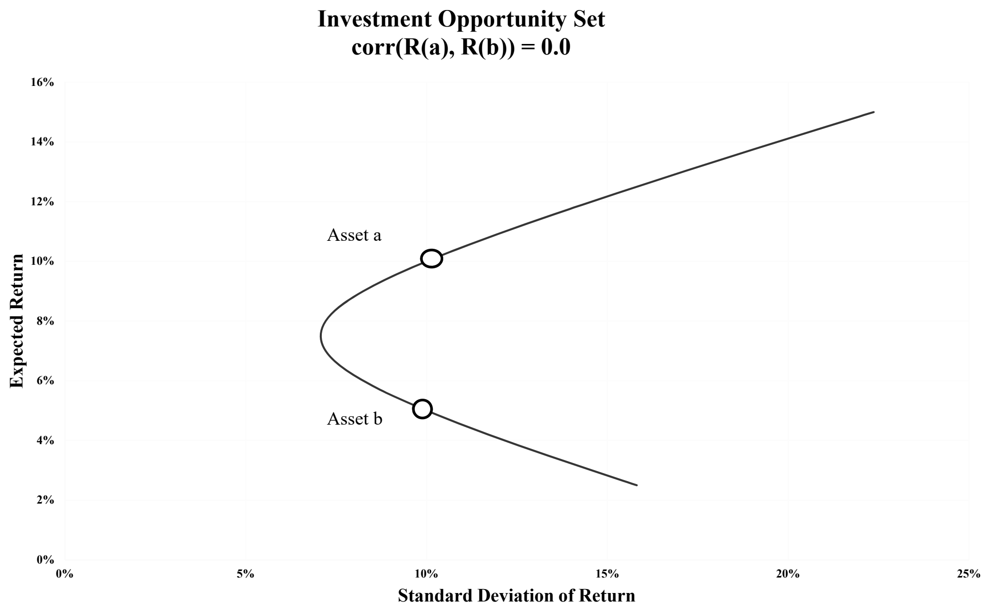
SD[R(a)] = SD[R(b)] =.10

You can create portfolios of a and b using different weights to get different possible combinations of expected returns and standard deviations

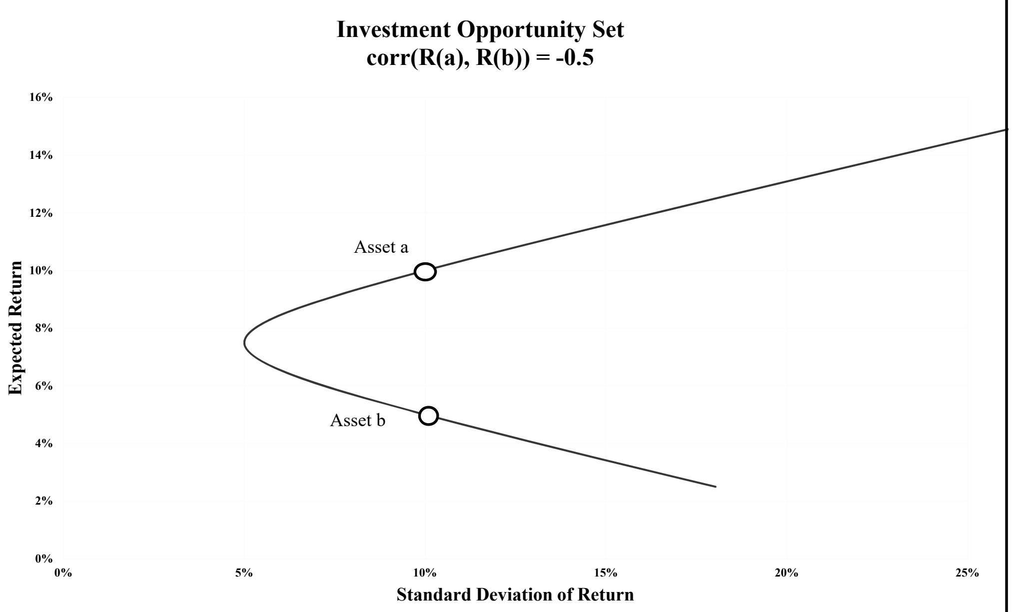
Investment Opportunity Set: 2 assets



Investment Opportunity Set: 2 assets



Investment Opportunity Set: 2 assets



Investment Opportunity Set: 2 assets

If the correlation between asset returns is perfectly negative (-1), then it is possible to create a combination of the two risky assets that is riskless:

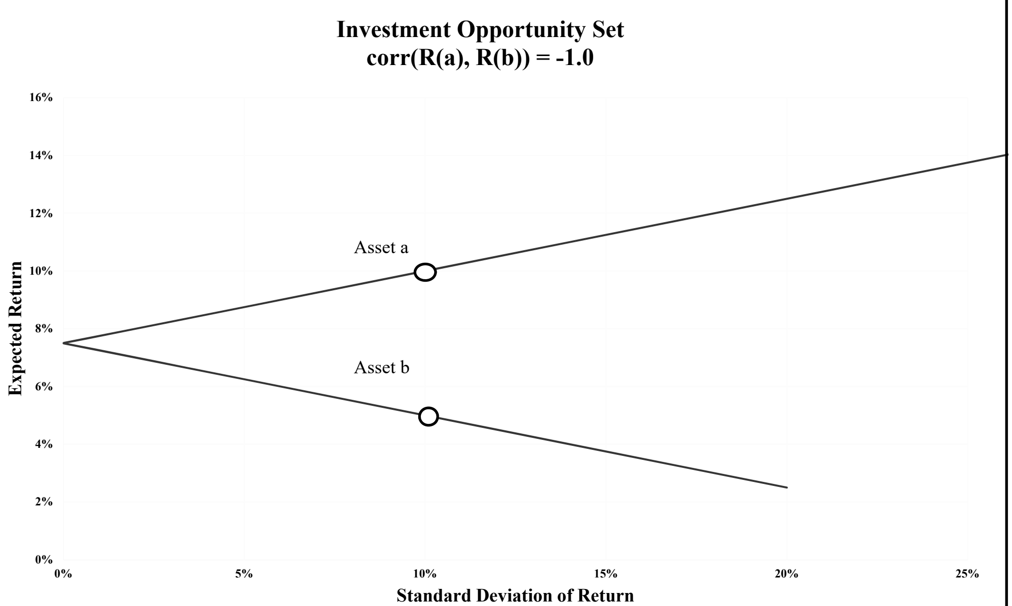
$$\begin{aligned} Var[R(pt)] &= w(at)^2\ Var[R(at)] + (1-w(at))^2\ Var[R(bt)] \\ &\quad - 2\ w(at)\ (1-w(at))\ SD\ [R(at)]\ SD\ [R(bt)] \\ &= [w(at)\ SD\ [R(at)] - (1-w(at))\ SD\ [R(bt)]]^2 \\ &= 0 \text{ when } w(at)\ SD\ [R(at)] = (1-w(at))\ SD\ [R(bt)] \end{aligned}$$

or

$$w(at) = SD\ [R(bt)]\ /\ [SD\ [R(at)] + SD\ [R(bt)]]$$

which in this case = ½ (i.e., half your money in each stock)

Investment Opportunity Set: 2 assets



Portfolio Theory in Practice

- It is very hard to get precise measures of expected returns from historical returns
- It is not hard to get precise measures of risk from historical returns
- If you try to use estimates of means and variances from historical data, the "optimal" portfolios you derive will load up on stocks that did well ex post in the measurement period
  - Which is unlikely to be repeated in the period when you are investing in the securities

Portfolio Theory in Practice

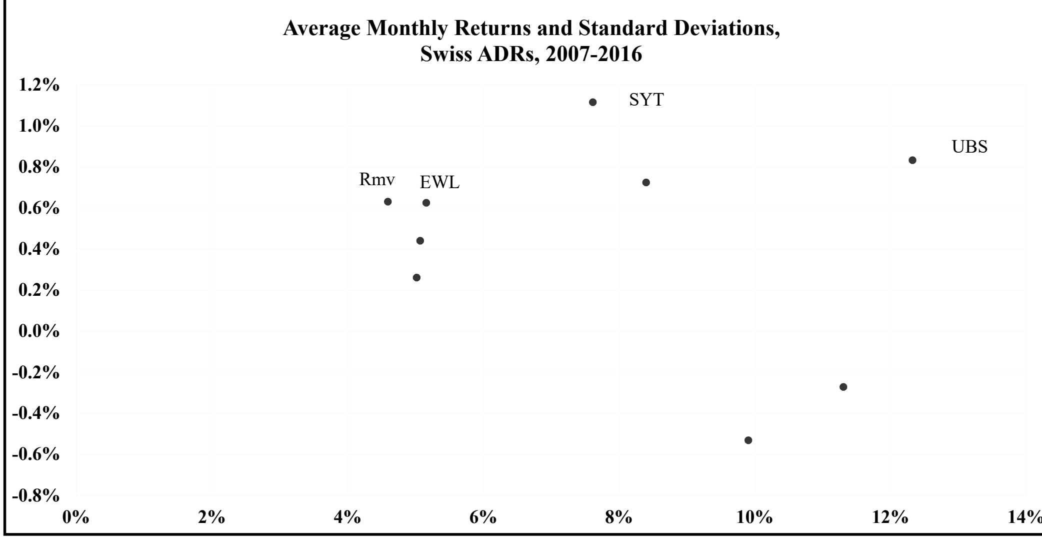
- A better approach is to form portfolios from "randomly selected stocks (so they are unlikely to be affected by common sources of risk/shocks)
- Problem with selecting stocks based on your knowledge of them is that they are likely to be affected similarly by:
  - Industry shocks
  - Geographic shocks, etc.
- From the perspective of an individual investor, investing in a pool (portfolio) of stocks may be a more economical method of achieving diversification
  - Mutual funds, ETFs, etc.

Investment Opportunity Set Example: 6 Swiss ADRs, 2 Sw Funds, Rmv, 2007-2016

Company	Ticker	Exchange	Industry
ABB	ABB	NYSE	Industrial Engineer.
Credit Suisse	CS	NYSE	Banks
Logitech	LOGI	NASDAQ	Tech.Hardware&Equip.
Novartis	NVS	NYSE	Pharma. & Biotech.
Syngenta	SYT	NYSE	Chemicals
UBS - Global Registered Shares	UBS	NYSE	Banks
The Swiss Helvetia Fund	SWZ		Closed end fund
iShares MSCI Switzerland Capped ETF	EWL		ETF that tracks the performance of the MSCI Switzerland 25/50 Index

Investment Opportunity Set Example: 6 Swiss ADRs, 2 Sw Funds, Rmv, 2007-2016

	Rmv	UBS	LOGI	NVS	SYT	ABB	CS	SWZ	EWL
Avg	0.63%	-0.27%	0.83%	0.63%	1.12%	0.72%	-0.53%	0.26%	0.44%
Std	4.59%	11.30%	12.32%	5.16%	7.61%	8.40%	9.90%	5.02%	5.07%



Portfolio Theory in Practice

- Rmv is the value-weighted portfolio of US stocks
- EWL is an exchange-traded fund (ETF) that mimics a large value-weighted portfolio of Swiss stocks (returns in US dollars)
- SWZ is a closed end fund traded in the US that holds Swiss stocks (basically a company that buys a portfolio of Siwss stocks as its assets), but the stock price is not mechanically tied to the prices of the underlying stocks
- UBS, LOGI, NVS, SYT, ABB, and CS are ADR's traded in the US linked to individual Swiss stocks (all large companies)

Portfolio Theory in Practice

- Note that the large portfolios have lower standard deviations given the level of average returns
  - This is what we mean by <u>DIVERSIFICATION</u>

Market Model Regression

- Regress return on asset i in time t on return on a "market" portfolio m in time t

$R(it) = \alpha_{im} + \beta_{im} R(mt) + e(it)$, t = 1, . . ., T

Where $\beta_{im} = cov([R(it), R(mt)] / var[R(mt)]$ is the "beta" coefficient

$\alpha_{im} = E[R(it) - \beta_{im} E[R(mt)]$

By definition, the weighted average $\beta_{im} = 1$ and the weighted average $\alpha_{im} = 0$, where the weights are the ones used to create R(mt) from a set of assets i

Market Model Regression

- Variance decomposition from market model regression:

$Var [R(it)] = \beta_{im}^2 Var[R(mt)] + Var[e(it)]$

Since it is assumed that the regression error is uncorrelated with the regressor (the market return)

Market Model Regression

Often-used definitions:

β_{im}^2 Var[R(mt)] = “systematic risk” or “market risk”

– can’t be diversified away

Var[e(it)] = “unsystematic risk” or “diversifiable risk”

– which is made smaller by creating a diversified portfolio

Market Model Regressions for Swiss Stocks Using Rmv, 2007-2016

Stock	a(i)	S[a(i)]	b(i)	S[b(i)]	Rsq	S[e]
UBS	-0.0127	0.0080	1.5800	0.1737	0.4120	0.0870
LOGI	-0.0015	0.0093	1.5559	0.2013	0.3362	0.1008
NVS	0.0029	0.0042	0.5318	0.0911	0.2242	0.0456
SYT	0.0064	0.0063	0.7462	0.1363	0.2026	0.0683
ABB	-0.0016	0.0050	1.4057	0.1077	0.5909	0.0539
CS	-0.0140	0.0071	1.3718	0.1532	0.4047	0.0767
SWZ	-0.0025	0.0031	0.8040	0.0680	0.5420	0.0341
EWL	-0.0013	0.0027	0.9045	0.0582	0.6719	0.0292

Note: Rsq = β_{im}^2 Var[R(mt)] / Var[R(it)] is larger and S[e] smaller for the portfolios SWZ and EWL

Betas for Swiss portfolios are less than 1 relative to Rmv (lower than average “systematic risk)

Market Model Regressions for Swiss Stocks Using EWL, 2007-2016

Stock	a(i)	S[a(i)]	b(i)	S[b(i)]	Rsq	S[e]
UBS	-0.0091	0.0079	1.4442	0.1565	0.4192	0.0865
LOGI	0.0034	0.0101	1.1146	0.1990	0.2101	0.1100
NVS	0.0029	0.0031	0.7714	0.0611	0.5744	0.0338
SYT	0.0075	0.0059	0.8196	0.1159	0.2977	0.0641
ABB	0.0015	0.0048	1.2997	0.0946	0.6152	0.0523
CS	-0.0108	0.0070	1.2505	0.1382	0.4095	0.0764
SWZ	-0.0014	0.0019	0.9052	0.0368	0.8365	0.0204

Note: Rsq = β_{im}^2 Var[R(mt)] / Var[R(it)] is larger and S[e] smaller for the portfolio SWZ

Betas for Swiss stocks are closer to 1 relative to the portfolio of Swiss stocks

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