BRN 481
Statistical Properties of Stock Prices

- We are going to talk about stock prices, dividends, and returns

- Facts (to come):
  - Stock returns are basically normally distributed variables that are random through time (but correlated across stocks at a point in time)
  - Stock prices, which are really just the accumulation of past returns, therefore behave like a “random walk”
  - While the path of the random walk looks like it has cycles and predictable patterns, in fact, future movements perhaps around a trend, are completely unpredictable
  - This will lead us to a form of the “efficient markets hypothesis”

Rates of Return

- $R(t) = \frac{[P(t) - P(t-1) + D(t)]}{P(t-1)}$
  - where $P(t)$ is price at time $t$
  - $D(t)$ is dividend paid at time $t$ to stockholder at time $t-1$

- This is close to the continuously compounded rate of return
  - $r(t) = \log (1 + R(t))$
  - as long as the return $R(t)$ is small (less than .15 in absolute value)
  - “$\log$” is the natural logarithm (the function $\ln(.)$ in Excel)
Rates of Return

• Simple returns add up (or average) across securities at a point in time
  – This is useful for understanding the returns to portfolios of securities

• Continuously compounded returns add up (or average) for a single security across different points in time
  – Useful for thinking about returns over different time horizons

Random Walk Model for Stock Prices

• If returns to stocks, \( r_t \), are random through time (unpredictable), perhaps because the market processes information efficiently and incorporates it into prices immediately,

• Prices (or the logs of prices) will follow a random walk, since this period’s (log) price, \( \log(P(t)) \), equals last period’s (log) price, \( \log(P(t-1)) \), plus this period’s random (continuously compounded) return (ignoring dividends for the moment):

\[
r_t = \log(P(t)) - \log(P(t-1)) = \log(P(t) / P(t-1)) = \log(1 + (P(t) - P(t-1)) / P(t-1))
\]
Random Walk Model for Stock Prices

- While the changes in (log) prices are random and unpredictable, the (log) prices in consecutive months contain much of the same information
  - The (log) price at time \( t \) is just the (log) price at time 0 plus the sum of all returns between 0 and \( t \)
    \[
    \log(P(t)) = r(t) + r(t-1) + \ldots + r(1) + \log(P(0))
    \]
  - So \( \log(P(t)) \) and \( \log(P(t-1)) \) share \( t-1 \) past returns, which means they will be highly correlated

Random Walks vs. Random Variables

- If changes in (log) prices are random, then (log) prices follow a random walk
- Difference between random variables (returns) and random walks (prices) is confusing for many students
Some Properties of Random Walks

• The mean and variance of returns are proportional to the length of the measurement interval $k$
  
  
  $E[\log(P(t)/P(t-k))] = k \mu$
  
  $\text{Var}[\log(P(t)/P(t-k))] = k \sigma^2$

• This is because the autocorrelations of returns are zero
  
  $\text{Corr}[r(t), r(t-k)] = 0$ for all lags $k$
  
  i.e., past returns do not predict future returns

Example: US Stock Market

• $Rm-Rf$, the excess return on the market, value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month $t$, good shares and price data at the beginning of $t$, and good return data for $t$ minus the one-month Treasury bill rate (from Ibbotson Associates).

  $\text{http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html}$
Example: Monthly US Stock Market Returns, 1926-2018

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Autocorrelations

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</table>

Example: Monthly US Stock Market Returns, 1926-2018

Histogram of US Market Excess Returns, 1927-2018

(c) Prof. G. William Schwert, 2017-2019
Example: US Stock Market

- Histogram of returns looks pretty “normal” (a few too many big negatives . . .)

- Also a few too many returns close to the middle of the distribution

Example: Monthly US Stock Market Returns, 1926-2018

Autocorrelations of US Market Returns, 12 Lags, 1927-2018
Example: US Stock Market

- Autocorrelations are small for 12 lags

- The apparent autocorrelation at lag 1 is due to “non-synchronous trading”
  - Not all stocks in the index trade at the end of the measurement period, so some price adjustment shows up in the next period’s return
  - Autocorrelations of return to individual stocks tend to be much smaller

Example: Monthly US Stock Market Returns, 1926-2018

Monthly Returns to US Stock Market, 1926-2018
Example: US Stock Market

- Time series graph of returns looks random (but it looks like dispersion is periodically high or low – more on this later)

Example: Monthly US Stock Market Returns, 1926-2018

Value of $1 Invested in US Stock Market, Starting in 1926
Example: US Stock Market

- Time series graph of the value of $1 invested in this portfolio, plotted on a log scale, looks like an upward trending random walk
  - Technical analysts would be tempted to find “patterns” in the graph to try to predict the future . . .

Example: Monthly US Stock Market Returns, 1926-2018

Volatility of Monthly Returns to US Stock Market, 1926-2018
Example: US Stock Market

- Graph of rolling 12-month standard deviation shows long periods of high or low volatility
  - Very low recently

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Data used for these slides can be accessed at:

http:schwert.ssb.rochester.edu\brn481\brn481rw.xlsx

http:schwert.ssb.rochester.edu\brn481\brn481rw.zip

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