Random Walk Model for Stock Prices

- If *returns* to stocks, r_t, are *random* through time (unpredictable), perhaps because the market processes information efficiently and incorporates it into prices immediately,
- Prices (or the logs of prices) will follow a *random walk*, since this period's (log) price, log(P(t)), equals last period's (log) price, log(P(t-1)), plus this period's random (continuously compounded) return (ignoring dividends for the moment):

$$r_t = \log(P(t)) - \log(P(t-1)) = \log(P(t) / P(t-1)) = \log(1 + (P(t) - P(t-1))/ P(t-1)))$$

Random Walk Model for Stock Prices

- While the changes in (log) prices are random and unpredictable, the (log) prices in consecutive months contain much of the same information
  - The (log) price at time t is just the (log) price at time 0 plus the sum of all returns between 0 and t

$$\log(P(t)) = r(t) + r(t-1) + \ldots + r(1) + \log(P(0))$$

  - So log(P(t)) and log(P(t-1)) share t-1 past returns, which means they will be highly correlated

Random Walks vs. Random Variables

- If changes in (log) prices are random, then (log) prices follow a random walk
- Difference between random variables (returns) and random walks (prices) is confusing for many students

Rates of Return

- R(t) = [P(t) – P(t-1) + D(t)] / P(t-1)
  - where P(t) is price at time t
  - D(t) is dividend paid at time t to stockholder at time t-1
- This is close to the continuously compounded rate of return
  - r(t) = log (1 + R(t))
  - as long as the return R(t) is small (less than .15 in absolute value)

Rates of Return

- Simple returns add up (or average) across securities at a point in time
  - This is useful for understanding the returns to portfolios of securities
- Continuously compounded returns add up (or average) for a single security across different points in time
  - Useful for thinking about returns over different time horizons

Some Properties of Random Walks

- The mean and variance of returns are proportional to the length of the measurement interval k
  - $E[\log(P(t)/P(t-k)] = k\ \mu$
  - $Var[\log(P(t)/P(t-k)] = k\ \sigma^2$
- This is because the autocorrelations of returns are zero
  - $Corr[r(t), r(t-k)] = 0$ for all lags k
  - i.e., past returns do not predict future returns

Example: US Stock Market

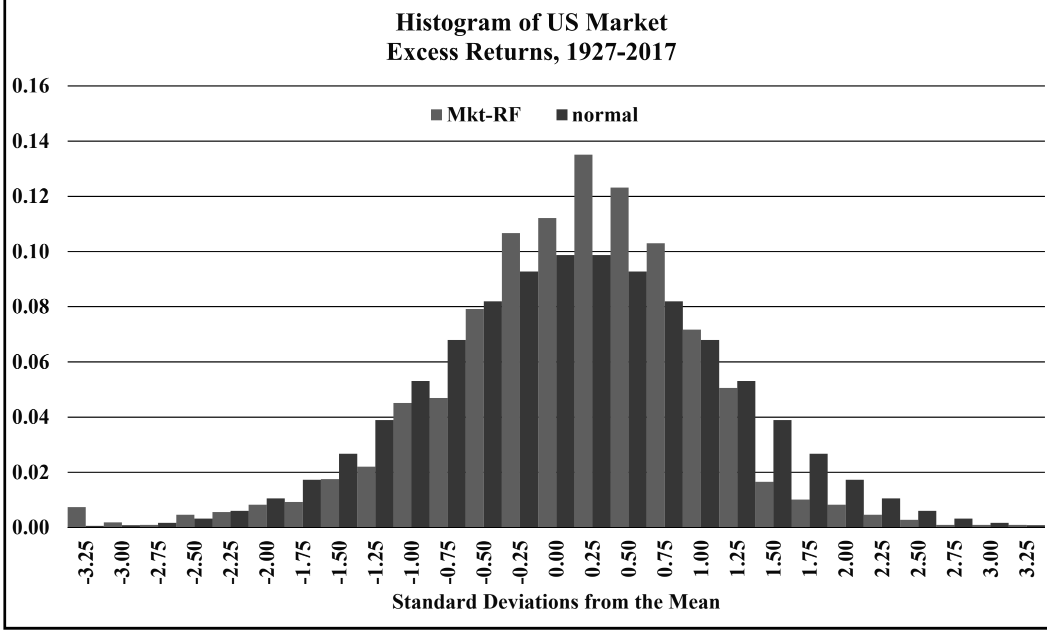
- Rm-Rf, the excess return on the market, value-weight return of all CRSP firms incorporated in the US and listed on the NYSE, AMEX, or NASDAQ that have a CRSP share code of 10 or 11 at the beginning of month t, good shares and price data at the beginning of t, and good return data for t minus the one-month Treasury bill rate (from Ibbotson Associates).
  - http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/f-f_factors.html

Example: Monthly US Stock Market Returns, 1926-2017

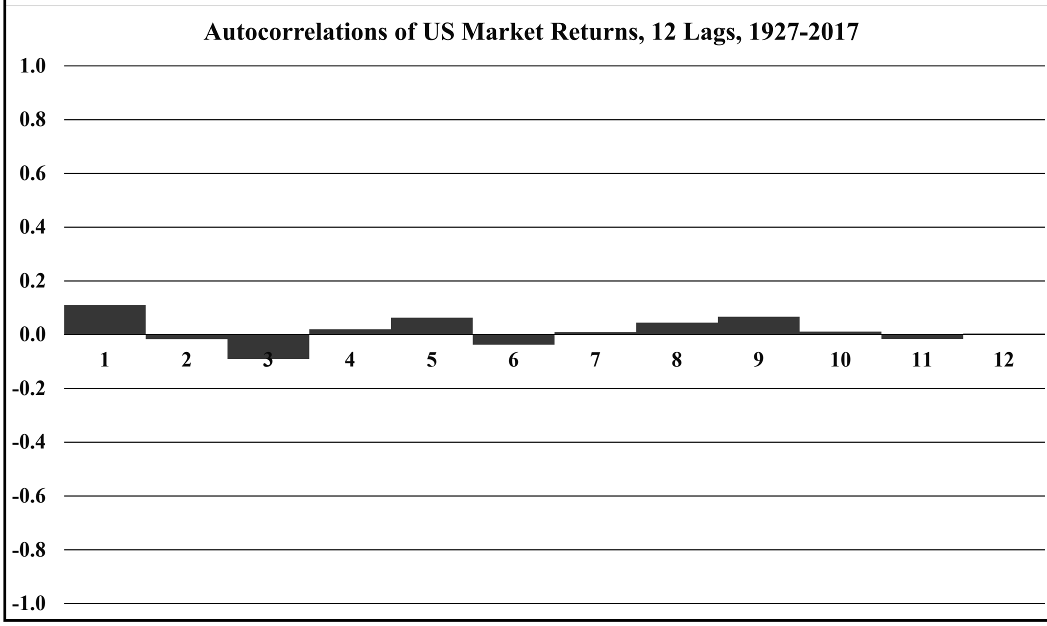
	Mkt-RF	12-month rolling sd
Avg	0.6563	4.6239
Std	5.3654	2.6794
Max	38.8500	20.5099
Med	1.0100	3.9861
Min	-29.1300	1.1326
N	1088	1077

Autocorrelations Lag	
1	0.11
2	-0.02
3	-0.09
4	0.02
5	0.06
6	-0.04
7	0.01
8	0.04
9	0.07
10	0.01
11	-0.02
12	0.00

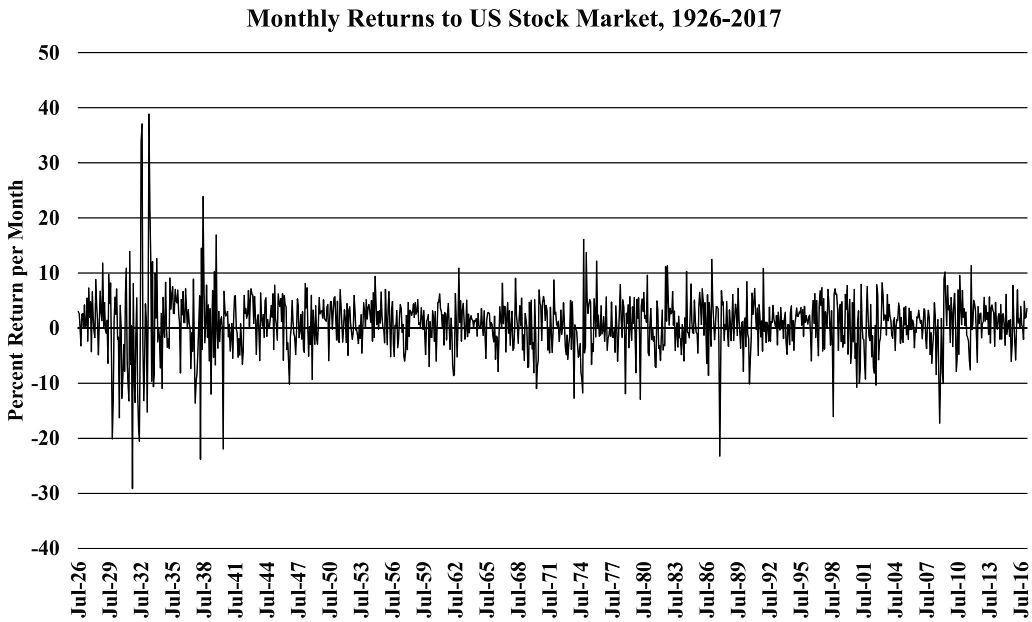
Example: Monthly US Stock Market Returns, 1926-2017



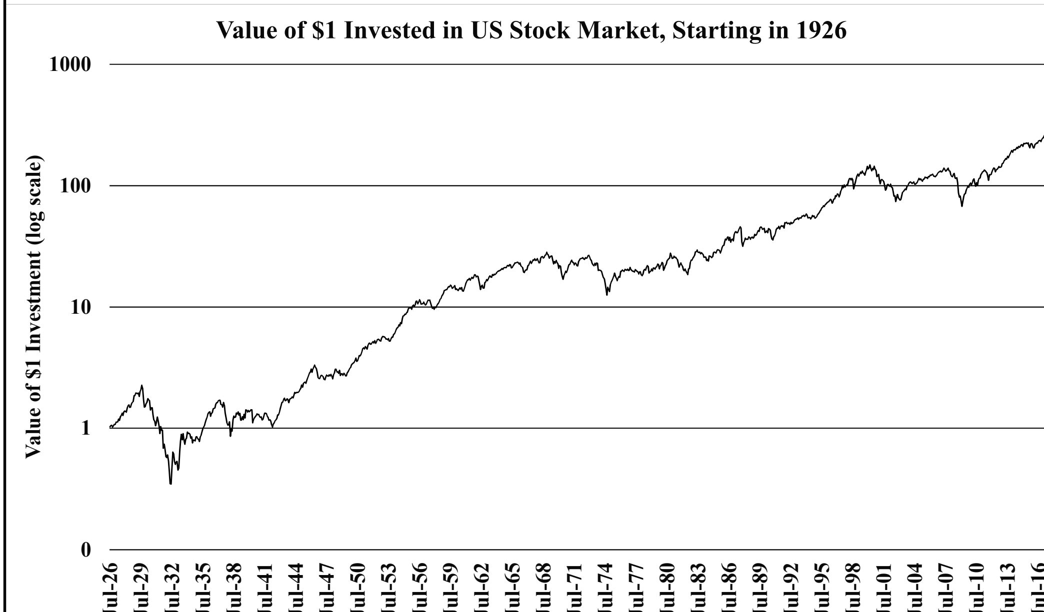
Example: Monthly US Stock Market Returns, 1926-2017



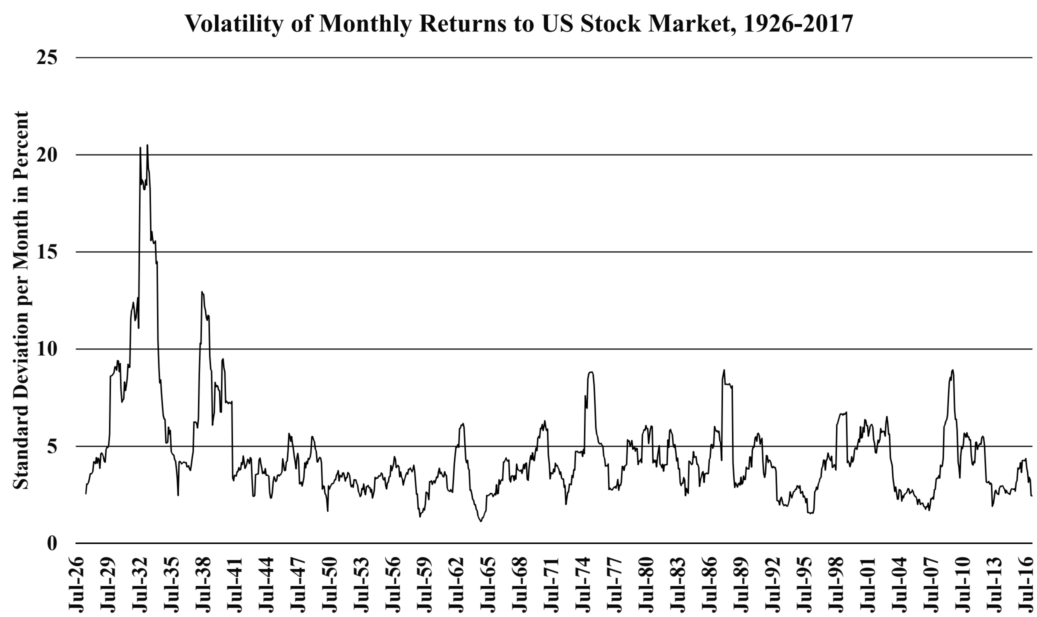
Example: Monthly US Stock Market Returns, 1926-2017



Example: Monthly US Stock Market Returns, 1926-2017



Example: Monthly US Stock Market Returns, 1926-2017



Example: US Stock Market

- Time series graph of returns looks random (but it looks like dispersion is periodically high or low – more on this later)
- Autocorrelations are small for 12 lags
- Histogram of returns looks pretty "normal" (a few too many big negatives . . .)
- Time series graph of the value of $1 invested in this portfolio, plotted on a log scale, looks like an upward trending random walk
  – Technical analysts would be tempted to find "patterns" in the graph to try to predict the future . . .
- Graph of rolling 12-month standard deviation shows long periods of high or low volatility

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Data used for these slides can be accessed at:

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