Building the IPO order book: underpricing and participation limits with costly information

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Abstract

We examine the book-building method for marketing IPOs. In our model, the underwriter selects a group of investors along with a pricing and allocation mechanism to maximize (at minimum cost) the information generated during the process of going public. We also address the moral hazard problem faced by investors when evaluation is costly. When there is little need for accurate pricing, the expected gain from underpricing exactly offsets the investors’ costs of acquiring information. When pricing accuracy is important, however, the number of investors participating in the offering is larger, underpricing is greater, and investors can earn economic rents.

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1. Introduction

Initial public offerings (IPOs) are marketed to investors in a number of different ways. An underwriter can offer the shares to the market after collecting only limited

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information, following certain “fairness” rules that give everyone an opportunity to participate in the offering.\footnote{1} This method had been used historically in most IPOs outside of the U.S. and is examined in articles by Rock (1986), Beatty and Ritter (1986), Chowdhry and Sherman (1996a, b), Brennan and Franks (1997), and Benveniste and Busaba (1997). In addition, various auction mechanisms have been widely used to sell IPOs. However, the largest IPOs in the U.S. and increasingly around the world are marketed in what is known as a book-building process, which we investigate in this paper.\footnote{2}

The book-building process, which was first examined in the academic literature by Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990),\footnote{3} can be described as consisting of three steps. The investment bank first decides which investors will be invited to evaluate and perhaps buy the issue. Next, investors evaluate the issue and provide the investment bank with preliminary indications of their demand for the issue. Finally, the investment bank prices the issue and allocates shares to investors, generally allocating more shares to investors who indicate higher levels of demand.

The book-building method is often criticized because, in contrast to the other methods, underwriters generally exclude investors from the bidding process. This practice of excluding investors is especially controversial because the shares are underpriced on average, giving at least the appearance that book building provides a convenient mechanism for underwriters to reward their favorite clients. The Benveniste and Spindt and Benveniste and Wilhelm articles suggest that there is an economic motivation for underpricing IPOs; specifically, investment banks need to underprice IPOs to effectively extract information from their investors. However, these papers do not explain why IPOs are marketed only to an exclusive group of investors. Indeed, in these models, exclusivity is costly since the investment bank can lower the level of underpricing by marketing the IPO to a larger group of informed investors.

\footnote{1}{Anyone is allowed to apply for shares in the public offer (also known as the fixed price, open offer, or universal offer) method, and orders are sometimes allocated pro rata. More often, small orders are favored over large orders. Chowdhry and Sherman (1996b) show that favoring small investors reduces the winner’s curse faced by the uninformed investor, thus reducing the amount of underpricing needed.}

\footnote{2}{Loughran et al. (1994) were the first to examine international patterns in the usage of various methods. Ljungqvist et al. (2001) and Sherman (2001) document the increased use of book building, the continued but declining popularity of public offers, and the relative rarity of auctions. The book-building and public offer methods are frequently combined in hybrid offerings where book building is used to set the price and to allocate shares to institutional investors, while a public offer tranche is reserved for retail investors. Another recent development is the sale of shares in five U.S. and two Australian IPOs via the internet, through uniform price auctions (sometimes mistakenly referred to as Dutch auctions). The American internet IPO auctions were handled by W.R. Hambrecht through their “open IPO” process (www.openipo.com), while those in Australia were through eCapital, part of Ord Minnett, using the process they call a “book build” (www.ecapital.com.au).}

Our model of the book-building process, an extension of earlier models, provides an explanation for why investment banks limit the number of investors who are invited to participate in an IPO. The model is based on two premises. The first is that collecting information is costly, which is a fairly standard and intuitive assumption. The second is that there are economic benefits associated with generating information in order that the IPOs are priced more accurately in the secondary market. This second assumption is consistent with the observation that investment banks pay top salaries for star analysts and do other things outside of the book-building process that help them accurately price IPOs.

The benefits of pricing accuracy can be justified in a number of ways. First, as discussed in Sherman (1992), Benveniste et al. (2002), Subrahmanyam and Titman (1999), and Maksimovic and Pichler (2001), there can be feedback from market demand to investment choices. Of course, the entrepreneurs in these models prefer having their firms’ shares sell for the highest possible price. However, the costs associated with mispricing (accepting negative NPV projects or being forced to abandon positive NPV projects) outweigh the benefits associated with selling overvalued stock. This implies that, ex ante, the entrepreneur would like a mechanism for going public that prices the firm as accurately as possible.

The better investment choices that arise as a result of more accurate pricing provide just one of many motivations for investment banks to value accurate pricing. In addition, Titman and Trueman (1986) suggest that investment banks will attract higher-quality clients if they maintain a reputation for accurate pricing. Their model suggests that the high-quality firms will pay a premium for a more accurate underwriter who can better certify that the firm is indeed high quality. Understandably, the lower-quality firm is not willing to pay a premium for a more accurate assessment of their lower value.

Similarly, Stoughton et al. (2001) point out that high-quality firms may be willing to pay for information acquisition in the stock market because they can benefit through a better-quality reputation in the product market. Accurate pricing of the initial offering might also be valued for liquidity reasons. When there is a substantial amount of private information that is not incorporated into a stock’s initial price, the stock is likely to be less liquid, which could adversely affect its value. Finally, it should be noted that both the issuer and the investment bank could be sued if a new issue is substantially mispriced.

As Benveniste and Spindt and others have emphasized, the investment bank’s pricing policy affects the incentives of investors to truthfully reveal their private information. In particular, if investment bankers do not sufficiently underprice those new issues with the most favorable information, investors will tend to underreport their information (i.e., they will downplay the value of the issue in an attempt to lower the issue price).

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4 The motivation of the book-building approach is to gather information to improve pricing accuracy. This should be contrasted with the rationale discussed in Rock (1986) for pricing IPOs so that the issue can be fully subscribed by uninformed investors. In Rock, issuers are risk averse and get no benefit from more accurate pricing. They thus want to minimize the amount of information revealed prior to the offering.
When information is costly for investors to obtain, there is a second rationale for underpricing new issues. Investors face a moral hazard problem because they know that the price of the issue will be based on all reported information. Rather than purchasing his or her own signal, each investor may be tempted to free-ride on the information of others. IPOs must be underpriced sufficiently to offset this problem and to induce all investors to collect information. In equilibrium, this information collection constraint, rather than the truth-telling constraint, is the binding constraint that determines pricing policy. We will show that if the pricing policy provides sufficient incentive for investors to collect information, they will also have sufficient incentive to reveal the information.

To understand this, note that the investment bank in our model collects more information if more investors participate, and that the only cost of attracting more participants is that the issue might need to be underpriced more. However, when only the truth-telling constraint binds, additional investors can be induced to participate without requiring additional underpricing. This is a Pareto-inferior solution, because additional information could be acquired at no cost. Thus, in equilibrium, the investment bank will always choose an investor pool large enough that the information purchase constraint is binding.

We also show that when there is little need for accurate pricing, relatively few investors will be invited to participate in the offering, relatively little underpricing will be needed, and the expected gain from underpricing will exactly offset the investors' cost of information. When there is a strong need for accurate pricing, more informed investors will be invited to participate in the offering. If enough investors are chosen, the informed investors will earn rents, i.e., the average underpricing will exceed their information costs. In addition, we show that exogenous limits on investor participation can lead to uninformed investors earning excess returns.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 discusses the various incentive constraints needed to induce investors to purchase and accurately report information, and describes the underwriter's optimization problem. Section 4 describes the equilibrium and presents the main results. Section 5 contains comparative statics, and Section 6 examines the effect of limiting investor participation. Section 7 discusses the circumstances under which investors earn abnormal returns, and Section 8 presents our conclusions.

2. The model

Our model assumes that there are no conflicts of interest between the underwriter, who will be pricing and marketing the issue, and the entrepreneur, who has chosen to take his or her firm public. Both prefer a high issue price but also value price accuracy for the reasons discussed earlier.

The model consists of three dates. At the first date, the investment bank announces the price and allocation schedule and selects the investors who will participate in the offering. The investors decide whether to purchase an information signal and choose whether or not to reveal the signal to the investment bank. The
investment bank then sets the offering price, based on the information provided by investors. At date 2 the shares begin trading, and at date 3 the state of the economy is revealed and the firm is liquidated. Below is a summary of the timing in this model.

<table>
<thead>
<tr>
<th>Date 1</th>
<th>Date 2</th>
<th>Date 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment bank selects investors, announces price and allocation schedule</td>
<td>Investors decide whether to purchase signals</td>
<td>Investors report signals to investment bank</td>
</tr>
<tr>
<td></td>
<td>Investment bank sets price and allocates shares (based on announced schedule)</td>
<td>Shares begin trading</td>
</tr>
<tr>
<td></td>
<td></td>
<td>True state revealed</td>
</tr>
</tbody>
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2.1. The issuer

The issuer requires a fixed amount of capital to initiate an investment project and plans to sell a fixed number of shares, \( N \). Holding \( N \) fixed allows us to avoid accounting for the different levels of dilution that would occur if the number of shares sold can vary. If the entrepreneur raises more than is needed to fund the investment (because the price is higher than anticipated), the excess will be paid to the original shareholders as a dividend.

We assume that the market price at date 2, the initial trading date, reflects the information gathered by the investment bank (through the price and allocations chosen), and that the true state, good (\( g \)), bad (\( b \)), or uncertain (\( u \)), is revealed and incorporated in market prices at date 3. At that date, the value per share given state \( j, j \in \{ g, b, u \} \), is \( s_j \). The value of the additional information is such that \( s^g = s^u + \alpha \) and \( s^b = s^u - \alpha \), where \( 0 < \alpha \leq s^u \).

2.2. The investors and their signals

To distribute and price the issue, the underwriter enlists the help of \( H \) risk-neutral investors, where \( H \) is chosen by the underwriter. These investors have access to capital as well as to information. As a departure from the existing literature, we assume that the investors must purchase their information, paying \( c \) dollars for a signal that may be good, bad, or uninformative (neutral). Good or bad signals should be interpreted as favorable or unfavorable information that the underwriter did not detect. To simplify our analysis we will assume that there can exist either favorable or unfavorable undetected information, but not both. This means that all investors who receive an informative signal receive the same signal.\(^{5}\) However, it is possible that some investors receive an informative signal while others receive an

\(^{5}\)One way to interpret the signal is as fundamental, verifiable information that has been overlooked by the investment bank during its due diligence evaluation. Another interpretation is that the investors are providing feedback on current market demand. However, demand for an issue depends on more than pure liquidity—fund managers do not buy a stock purely because they have some spare cash. Demand depends on the value that investors expect the market to place on the stock, now and in the future.
uninformative or neutral signal. This information environment is consistent with Benveniste and Wilhelm (1990).

Investors have an alternative investment with an expected return of \( r \), which to simplify notation is set equal to zero. In another departure from the existing literature, our model does not impose binding wealth constraints on the investors. The institutional investors in IPOs manage large amounts of money, relative to the size of the IPOs, and thus liquidity constraints are unlikely to be the primary determinant of their allocations. As long as the return is sufficient, investors will purchase as many shares as the underwriter allocates to them. In Section 6 we consider the effects of a wealth constraint.

2.3. Allocations, prices, and probabilities

In addition to selecting the number of investors, the underwriter prices the issue and decides how many shares to allocate to each investor. The allocation to each investor will depend on both the signal reported by that particular investor and the signals reported by other investors (i.e., on \( h \), the number of investors out of \( H \) that report either \( g \) or \( b \)). The following notation is for all \( j \in \{g, b, u\} \) and all \( k \in \{g, b\} \):

\[
\begin{align*}
    s_j & = \text{average issue price when state } j \text{ is reported (recall that } s_j \text{ is the share value in state } j); \\
    q_{j,k,h} & = \text{allocation to an investor who reports } j \text{ when } k \text{ is reported by } h \text{ of } H \text{ investors}; \\
    q_{j,k} & = \text{expected allocation to an investor who reports } j \text{ when at least one investor reports } k; \\
    q_{k,k}^* & = \text{expected allocation to an investor who reports } k, \text{ when at least one other investor also reports } k \text{ (this average excludes either } q_{g,g,1} \text{ or } q_{b,b,1}); \text{ and } \\
    q_u & = \text{allocation per investor when all investors report } u.
\end{align*}
\]

Last, we must specify the probabilities. Let \( \pi \) be the probability that there exists relevant information that was overlooked by the underwriter. We assume that the

(footnote continued)

Therefore, our view of the information being gathered is that it involves not only the verification of objective facts but also the interpretation of those facts. Valuation is a complex process that includes evaluating the capabilities of management and forecasting the future of the firm, the industry, and the economy. Experienced fund managers who have read the prospectus, heard the sales presentation, and questioned the management of the company may in the end know “more” than the investment bankers about the market value of the issue.

Last, one should note that the issue price and number of shares sold depend on the feedback from investors during book building. IPOs have even been canceled as a result of the response to a road show. Therefore, the underwriters involved obviously feel that they are getting some sort of information from the process, however one wants to interpret that information.
information that was overlooked is equally likely to be good or bad. With probability \((1 - \pi)\) there is no additional information for the investors to discover.

Let \(\pi_i\) be the conditional probability that investor \(i\) receives an informative signal, either good or bad (conditional on the existence of overlooked information). The unconditional probability that investor \(i\) will receive a good (or bad) signal is \(\pi_i\pi/2\). The unconditional probability of state \(g\) occurring and of \(h\) of the \(H\) investors receiving the signal \(g\) is \(P(g, h)\). Since the probabilities are symmetric, \(P(b, h) = P(g, h)\). The unconditional probability that none of the \(H\) investors receive informative signals is \(P(0, 0) = P(g, 0) + P(b, 0) + (1 - \pi)\).

The conditional probability that \(h\) of \(H - 1\) other investors will receive a good signal, given that one investor has received a good signal, is \(P'(g, h)\), and again \(P'(b, h) = P'(g, h)\). Finally, given that one investor fails to receive an informative signal, the conditional probability that no other investor will receive an informative signal is

\[
P'(u, 0) = \frac{1}{1 - \pi \pi_i} \left[ 1 - \pi + \frac{\pi}{2} (1 - \pi_i) (P'(g, 0) + P'(b, 0)) \right]. \tag{1}
\]

3. Incentive constraints and the underwriter’s objective function

There are two main types of incentive constraints for investors in this model. The investment bank needs to give investors the incentive both to buy the information and to report it accurately. Section 3.1 examines the information reporting or truth-telling constraints, while Section 3.2 examines the information collection constraints. Section 3.3 gives the investment bank’s maximization problem.

3.1. Information reporting constraints

As part of its book-building strategy, the underwriter must design an allocation and pricing schedule that elicits accurate information from investors. Since the investment bank will use the reported information to price the issue, the pricing and allocation strategy must counteract investor incentives to withhold favorable information that will lead to a higher issue price. We will be considering Nash equilibria where, conditioned on the underwriter’s strategy, investors have an incentive to truthfully reveal their information, given their expectation that other investors will also report information accurately.

There will typically be multiple solutions to the underwriter’s problem in this model, with many sets of allocations and prices that elicit truthful revelation and give the underwriter the same expected utility. From both the investors’ and our point of view, many of these multiple equilibria are effectively the same. To eliminate extraneous equilibria, we make the following assumptions:

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6The formulas for \(P(g, h)\) and \(P'(g, h)\) are given in Benveniste and Wilhelm (1990) and Sherman (2000).
1. The underwriter allocates zero shares to all investors who reveal conflicting signals. For example, if one investor reports $g$ and another investor reports $b$, the underwriter knows that someone is lying, although it cannot tell who, so it allocates zero shares to both. Although these conflicting reports will not exist within the equilibrium, this out-of-equilibrium assumption is needed to specify the equilibrium.

2. Investors who provide the same signals receive the same allocations.

3. The price per share in states $g$ and $b$ does not depend on $h$, the number of investors who report an informative signal.

Because investors are risk neutral, the issuer has no incentive to violate assumptions 2 or 3. Varying either allocations or prices based on $h$ would not change the expected returns that investors require; it would simply increase the variance of investor returns. An exception to this is when there are binding participation limits on investors. As we show in Section 6, the underwriter can lessen the effects of certain exogenous investment limits by making prices a function of $h$.

Let $R(j,k)$ be the expected profit to an investor who receives signal $j$ and reports signal $k$. In equilibrium, investors are induced to report their information truthfully, which implies that the following truth-telling constraints (described in more detail in Appendix A) must be satisfied:

$$R(j,j) \geq R(j,k) \quad \text{for all } j,k \in \{g,b,u\}. \quad (2)$$

It should be noted that the cost of acquiring information does not affect the information reporting conditions, since it is a sunk cost by the time the investor decides what signal to report. On the other hand, whether or not the investor plans to accurately report the signal certainly affects the incentive to buy a signal. After all, if the investor planned to report $u$ (or $g$ or $b$) regardless of the actual signal, then there would be no reason to buy a signal.

### 3.2. Information collection constraints

In addition to the truth-telling conditions, a constraint is needed to guarantee that investors choose to acquire information. The condition is that buying and reporting a signal offers at least as high an expected profit as not purchasing a signal and falsely reporting $u$:

$$(\pi_2\pi/2)\{R(g,g) + R(b,b)\} + (1 - \pi_2\pi)R(u,u) \geq R(\cdot,u) + c,$$

where $R(\cdot,u)$ is the expected return to an investor who reports $u$ without observing a signal. This expression can be rewritten as

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7 Similarly, because investors are risk neutral and because they make their decisions without knowing $h$, we can look at the average allocations in each situation without solving explicitly for the price and allocation for each investor given every possible $h$. (Additional information on how the underwriter will set individual allocations is available on request from the authors.)
In addition, buying a signal and reporting it must be at least as good as saving the cost $c$ and falsely reporting either $g$ or $b$ (or not participating at all). As long as the truth-telling constraints are satisfied, however, the return to falsely reporting $g$ or $b$ (or to not participating) will never be higher than the return to falsely reporting $u$.

3.3. The underwriter’s objective

The underwriter has nine main choice variables: the number of investors to include; three prices (one for each state); and five allocations, including one each for investors who report $g$ or $b$ and three for investors who report $u$ (one for each state). These variables are chosen to maximize a utility function that is a separable function of the accuracy of the initial aftermarket price and the expected amount by which the shares must be underpriced. In particular, the underwriter chooses $H$, the number of investors invited to participate in the offering, by trading off the increase in accuracy associated with a larger group of investors against a corresponding increase in the expected amount of underpricing. To illustrate this tradeoff, we first solve the underwriter’s maximization problem, as given below:

$$
\text{Max} \quad \left\{ (\pi/2) - P(g, 0) s_g + ((\pi/2) - P(b, 0)) s_b \right. \\
+ P(\cdot, 0) s_u - f(P(b, 0)) \right\}
$$

subject to Eqs. (2) and (3) and the following constraints that guarantee that $N$ shares are sold regardless of the state:

- $h q_{j,h} + (H - h) q_{u,j,h} = N, \quad \text{for all } 0 \leq h \leq H, \quad j \in \{g, b\};$
- $H q_u = N.$

The underwriter is maximizing the expected price per share, minus a term that reflects the possibility of price inaccuracy. The function $f(P(b, 0))$ is included in the utility function to capture the underwriter’s desire for greater pricing accuracy. $P(b, 0)$ is the probability that a bad (or equivalently a good) state exists that has not been discovered. We assume that $f(P(b, 0)) > 0$ for all $P(b, 0) \in (0, \pi/2)$, $f'(\cdot) > 0$, $f''(\cdot) > 0$, $f(0) = 0$, and $f(\pi/2)$ is sufficiently large, relative to the cost of the information, that the underwriter will always choose to induce at least one investor to purchase a signal.8

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8. If the underwriter and the issuer do not value price accuracy, then the optimal solution will be to allow an unlimited number of investors to participate in the offering. Each investor would be expected to receive such a small number of shares that it would not be cost-effective to evaluate the issue. The shares would be priced at their expected value minus some arbitrarily small but positive amount, and we would see few or no restrictions on investor access to shares and essentially no underpricing.
4. The equilibrium

The following analysis characterizes a Nash equilibrium in which each investor selected by the underwriter optimally purchases information and truthfully reveals the information to the underwriter. As we will show, this equilibrium requires that the underwriter use specific pricing and allocation rules that are optimal within the context of our model. The general properties of the equilibrium are described in Proposition 1, the proof of which is in Appendix C.

**Proposition 1.** Given our assumptions, the solution to our model will have the following properties.

- Good or “hot” (state \( g \)) issues will always be underpriced.
- Neutral (state \( u \)) or bad (state \( b \)) issues will be underpriced only when \( H \), the number of investors, is high (i.e., when the underwriter places a high value on price accuracy).
- For a sufficiently high \( H \), all issues will be underpriced.
- Allocations to informed investors of shares in hot issues will be higher for those investors that express a stronger interest (\( q_{g,a} > q_{u,a} \)).

The first and last properties—that hot issues are the most underpriced and that investors expressing more of an interest tend to get more shares—are an essential element of all book-building models. These are the properties that make investors willing to reveal favorable information that increases the offer price. However, earlier models have not considered the truth-telling constraints against overreporting (i.e., against exaggerating the quality of the issue in order to receive a larger allocation) and thus have not recognized that \( H \) can sometimes be too high to allow underpricing of only the best issues. As we show in the proof of Proposition 1, the overreporting constraint, Eq. (A.4) in Appendix A, affects which issues are underpriced.

Moreover, previous book-building models have not endogenized the choice of \( H \) and thus have focused on the truth-telling constraint against underreporting hot issues (i.e., against withholding good information). When there is a cost to information and when the underwriter controls the size of the investor pool, this constraint does not bind in equilibrium. Since there is always, in practice, at least some cost to evaluating an issue, and since control of investor allocations is the distinguishing feature of the book-building method, it is important to consider the constraint that binds in these circumstances, which is the information purchase constraint.

This result—that underpricing will be determined by the need to induce investors to acquire information—is formally stated in Proposition 2. The intuition behind the result is that as the marginal value of one investor’s information decreases, it becomes easier to induce that investor to accurately report the information but harder to induce him or her to pay to acquire it. The benefit of withholding good
information is that it can lead the issue to be even more underpriced than it otherwise would be. The lower the marginal value of the information withheld, the lower is the resulting increase in underpricing and hence the lower is the benefit of withholding. As $H$ increases, the marginal value of an individual signal decreases relative to the $H - 1$ other signals, and so less underpricing is needed to induce truth-telling.

On the other hand, the benefit of not acquiring information is that the investor saves the time and effort involved (represented in our model by a fixed cost, $c$). If all other investors evaluate and report truthfully, then any individual investor can free-ride by simply reporting “$g$”. A favorable report guarantees that the investor will receive an allocation of underpriced shares, assuming that the shares truly are high valued. The main cost of this strategy is that reporting a false good signal might lead to the issue being overpriced. However, the more total signals the underwriter is collecting, the lower is the probability that one false signal will lead to overpricing, and thus the greater is the incentive to avoid incurring the fixed cost $c$. Therefore, as $H$ increases, more underpricing is needed to induce information collection.

A larger investor pool directly benefits the issuer through greater price accuracy. If the information purchase constraint were not binding, then an increase in $H$ would increase accuracy and actually decrease underpricing, which would clearly make the issuer strictly better off. This implies that the issuer will continue to increase $H$ until the information purchase constraint binds. This intuition is proven in Proposition 2. This and other features of the solution are demonstrated in a numerical example in Appendix B, while the proof of the proposition is given in Appendix C.

**Proposition 2.** For low $H$, the number of investors, underpricing can decrease as $H$ increases, whereas, for sufficiently high $H$, underpricing always increases as $H$ increases. In equilibrium, the underwriter will always select an investor pool large enough that underpricing increases if $H$ is increased further.

From Proposition 1, neutral and/or cold issues are only underpriced when the issuer’s demand for price accuracy (and therefore its choice of $H$) is sufficiently high. In these cases, investors are able to earn excess returns. For lower $H$, investors earn only enough to compensate them for their information costs, leaving them indifferent as to whether or not to participate in the offering. However, to overcome the free rider problem when $H$ is very high, investors must be paid an excess return, which in turn means that all investors will strictly prefer to participate in the offering. This leads us to the following proposition, which is also proved in Appendix C.

**Proposition 3.** When the underwriter places a sufficiently high value on accuracy, informed investors will earn economic rents and participation in the IPO will be rationed.
5. Comparative statics

The results of the last section illustrate the importance of costly information in selecting the optimal size of the investor pool, as well as the importance of making the size of the investor pool endogenous. If information is free \((c = 0)\), then constraint (3) never binds. In this case, the information reporting constraints determine underpricing, and underpricing decreases as the number of investors \(H\) increases. The optimal \(H\) would be infinite, meaning that underwriters would not limit the number of investors in an IPO.

In reality, becoming informed is costly and underwriters regularly limit the number of investors that have access to new issues. As we show in the following propositions, which are proved in Appendix C, the number of investors given access to a new issue depends on the cost of collecting information and the value of that information.

**Proposition 4.** As \(c\), the cost of buying a signal decreases,

- the size of the investor pool increases, and
- underpricing of hot issues (where state \(g\) is revealed) decreases.

**Proposition 5.** As the value of the information (i.e., \(\pi\), the probability that a good or bad state exists) or the value that the underwriter places on pricing accuracy (i.e., \(f'(P(b,0))\)) increases,

- the size of the investor pool increases, and
- average underpricing increases.

These propositions state that as information becomes either more valuable or less expensive, the underwriter, in effect, purchases more of it by increasing the investor pool. As the size of the investor pool increases, expected underpricing must increase to cover the cost of the additional information, since expected total underpricing must equal the total cost of the information. However, as the following proposition demonstrates, underpricing of hot (state \(g\)) issues actually decreases as information becomes less expensive and it can also decrease as information becomes more valuable.

**Proposition 6.** As the accuracy of the information (i.e., \(\pi_i\), the probability that the signal will be informative) increases,

- the size of the investor pool can either increase or decrease, and
- underpricing decreases.
The second result, that underpricing decreases, arises because it is easier to convince investors to purchase a signal when that signal becomes more accurate. The first result is ambiguous, since there is both an “income” and a “substitution” effect with an increase in signal accuracy. When information is very accurate (high $\pi_i$), the number of participants is small because extra investors are not needed to achieve a high degree of price accuracy. When the accuracy of each signal is low, on the other hand, more signals are needed to have a reasonable chance of discovering bad (or good) issuers. However, if accuracy is too low, signals will simply not be worth purchasing. At very low accuracy levels, an increase in accuracy can actually lead to an increase in the number of investors, particularly if $p$ and $f''()$ are low also (i.e., if the value of information is small and the preference for greater price accuracy increases only at a slow rate).

Ljungqvist et al. (2001) compare data on book building and public offer IPOs for a large number of countries. They find that book building leads to lower underpricing only when it is conducted by U.S. banks and/or targeted at U.S. investors. Their results are consistent with our model if U.S. underwriters, analysts, and institutional investors are more efficient in evaluating offerings, perhaps because of the greater competitiveness of U.S. financial markets and the high regulatory standards on information revelation in the U.S. Propositions 4–6 demonstrate that underpricing will be lower when (1) the value of information is reduced (because the U.S. investment banks’ analysts have already evaluated the issue, substantially reducing the remaining uncertainty) and/or (2) the cost of information is reduced or the accuracy is increased (because U.S. institutional investors are more efficient and effective at evaluating IPOs).

6. The effects of participation limits

This section considers the effects of limits on investor participation. So far, we have not imposed any fixed limits on either the number of shares that an investor can purchase or the number of investors participating in the offering. We will now consider the effect of a minimum number of investors or a maximum allocation per investor. More specifically,

I. there must be at least $H^*$ investors purchasing a minimum of $e$ shares each; or
II. there is a maximum number of shares that can be allocated per investor, $q^\text{max}$.

The first limit is imposed in some countries to ensure broad participation by investors. For instance, in Australia there must be at least 500 shareholders each holding shares worth at least AUD 2000. The second limit might exist because of either cash constraints or limits in the mandate of a mutual or pension fund (e.g., some funds require that no more than a certain percentage of their portfolio can go into any one stock). Alternatively, the limit could be due to a law requiring investors to report their holdings when they have more than, say, 5% of total shares, or to
make a general offer if they hold more than, say, 35%. Finally, either of these constraints can arise because of corporate control considerations.

Assuming that the restrictions are binding, the size of the investor pool must increase. As $H$ increases, underpricing will increase also. The underwriter is trying to convince more investors to purchase signals and will have to offer a lower price to get them to do so. Thus, we have the following result, which is proved in Appendix C:

**Proposition 7.** With restrictions on the minimum number of investors or the maximum allocation per investor, both expected underpricing and the size of the investor pool will increase.

In addition to increasing underpricing, these restrictions make it necessary for the underwriter to allocate at least some shares in state $g$ issues to investors that report $u$. In order to minimize the cost of doing this, the underwriter will shift underpricing to just a few special cases where virtually everyone reports a good signal. Good issues will be fairly priced whenever $q_{u,g,h} > 0$, with underpricing concentrated in the cases where $q_{u,g,b} = 0$. Thus, we would expect IPO initial returns to be highly skewed, with most returns relatively low but with a few hot issues having enormous price jumps.

A skewed distribution pattern of initial returns can result from either type of participation restriction. However, other distribution effects on initial returns will depend on which type of restriction occurs. The first type of limit (a minimum number of investors) is more likely to affect small issues, implying that underpricing will be greater for small than for large issues. The second type of limit (a maximum allocation per investor) is more likely to affect large issues, implying the opposite size pattern for initial returns. When the limits occur for reasons of corporate control, the pattern will only be evident for firms that have reason to fear loss of control (for instance, when there is no controlling shareholder).

An alternative way of satisfying the above constraints is to allocate shares to uninformed investors. In Benveniste and Wilhelm’s model, uninformed investors are useful because they increase the underwriter’s ability to induce truth-telling from informed investors. This occurs because the underwriter has the option to distribute shares to uninformed investors when the informed investors give less favorable signals. In our model, there is no incentive to do this because, in equilibrium, it is the information collection constraint that binds.

Nevertheless, if there are binding participation restrictions, uninformed investors do indeed become useful. The restrictions force the number of investors $H$ to

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9 In Section 3.1, we made some simplifying assumptions to eliminate uninteresting, extraneous equilibria; in this section we are relaxing the assumption that prices do not depend on $h$, since making $s_p$ a function of $h$ now makes a significant difference in the solution.

10 U.S. firm commitment underwriters allocate IPO shares to both institutional and retail investors. One rationale for including retail investors is that the retail brokerage offices provide information about the retail demand for an issue which is valuable for pricing the issue. A second interpretation is that the retail investors are uninformed and are included to broaden the distribution of the shares for regulatory or corporate control reasons. Much IPO research assumes that retail investors are uninformed.
increase, which leads to inefficient levels of information collection and to excessive underpricing. Being able to satisfy the restrictions partly through the addition of uninformed investors allows the underwriter to limit the increase in underpricing and information gathering. However, the uninformed investors only help the underwriter up to a point, since replacing too many of the informed investors with uninformed investors will mean giving excess returns to the new investors without receiving any additional information in exchange. The underwriter will always benefit from inviting at least one uninformed investor to participate in the offering, but may or may not choose to add more than one such investor. This is discussed in the following proposition, which is proved in Appendix C.

**Proposition 8.** Given limits on investor participation, the underwriter will always be able to reduce underpricing by replacing at least one informed investor with an uninformed investor.

7. **When do investors earn economic rents?**

In the previous literature, e.g., Benveniste and Spindt (1989) and Benveniste and Wilhelm (1990), investors are endowed with information and thus enjoy rents whenever they are allocated underpriced shares. In our model, the cost of collecting information can offset an investor’s gain from underpricing. In some cases, the expected level of underpricing exactly offsets the cost of information collection, which implies that there are no rents associated with being selected by the underwriter to participate in a new issue. However, in other cases, the level of underpricing will exceed the information costs. Our model suggests two reasons that this may be the case. The first relates to the issuer’s desire for price accuracy. The second has to do with the participation restrictions described in the previous section.

First, excess returns are more likely if the issuer places a high value on price accuracy. When the demand for accuracy is sufficiently great, offering a return large enough to compensate for information costs requires that shares be underpriced in neutral and/or bad states as well as in good ones, as shown in Proposition 3. In these cases, there will be rents associated with being invited to participate in an IPO.

In a relevant empirical study, Hanley (1993) finds that issues priced within their initial filing range had an average initial return of 10.0%, while those priced strictly below the bottom of the price range listed in their prospectus had a positive but statistically insignificant average return of 0.6%. For IPOs priced strictly above their initial filing range, the average initial return was 20.7%. The differences in initial returns between these three categories are significant at the 1% level. Given the 10.0% initial return for “medium” IPOs (i.e., neither hot nor cold, the equivalent of state $u$ issues in our model), our model implies that informed investors are indeed earning excess returns on average for U.S. IPOs.
Another consequence of a strong desire for price accuracy is that $q_{u,g}$ can be positive. For $H$ sufficiently low that Eq. (A.4), which guarantees that investors observing $u$ do not falsely report $g$, does not bind, it is optimal to set $q_{u,g} = 0$, meaning that informed investors receive shares in hot IPOs only if they express a strong preference for them. In our model, $q_{u,g} > 0$ is another sign (besides the underpricing of $u$ and/or $b$ issues) that Eq. (A.4) is binding. Thus, if we see informed investors receiving at least some shares in all offerings by an underwriter, our model implies that those investors are indeed earning economic rents. Such a pattern is shown in Hanley and Wilhelm (1995) and James (1997).\[10\]

The second reason that investors might earn excess returns is because of the participation restrictions discussed in the last section. If firms require a broad shareholder base, either because of listing requirements or for corporate control reasons, we expect to see more underpricing. Informed investors will still receive just enough to compensate for their evaluation costs (unless the desire for price accuracy is very high), but uninformed investors might receive abnormal returns. As shown in the proof of Proposition 8, uninformed investors receive excess returns only as a response to restrictions on the minimum number of investors, and not in response to a limit on the maximum allocation per investor.

So far, we have assumed that all shares are sold at the same price. Benveniste and Wilhelm (1990) were the first to analyze the “one price rule” that is imposed in most countries, including the U.S., prohibiting an issuer from charging different prices to different investors. The rule does not bind in this model unless there are participation restrictions. With a binding restriction on the minimum number of investors, however, the issuer would sometimes benefit from being able to charge uninformed investors a higher price. This would allow the underwriter to compensate informed investors for their information acquisition without giving excess returns to free riders. The combination of a one price rule and a binding minimum number of investors thus leads to greater underpricing and to excess returns for uninformed investors.\[12\]

\[10\] Hanley and Wilhelm document allocation patterns for U.S. firm commitment IPOs by a major underwriter. This type of published study is rare, because investment banks prefer to keep their data confidential. Hanley and Wilhelm find that the institutional investors on the underwriter’s list typically take a share in every IPO by that underwriter (as long as the issue matches the strategy of the fund). James looks at 13f filings of institutional investors in the first quarter of each new issue to see which institutional investors have shares. He finds evidence that investment banks form stable coalitions with groups of investors who participate regularly in that underwriter’s issues. Hanley and Wilhelm find first quarter 13f holdings to be highly correlated with initial allocations, although there may be some bias. Benveniste et al. (1998) and Krigman et al. (1999) use TAQ data to show that institutional investors have a tendency to flip or sell cold issues in the first day of trading. If anything, this would bias James’ study against finding steady investor groups, yet such groups are still apparent.

\[12\] Many countries require the use of a hybrid method for IPOs: book building is used to set the offering price and allocate to institutional investors, while a public offer tranche is reserved for retail investors. The effects of this requirement are similar to the effects of requiring a minimum number of investors: underpricing will increase, and uninformed investors will earn excess returns. This occurs because the allocation rules of the public offer tranche, combined with the rule that all shares must be sold at the same price, prevent the issuer from using the underpriced shares to induce information acquisition and instead force the issuer to give part of the shares to free riders. This effect of hybrids is in addition to the problem
A restriction on the minimum number of investors is a common listing requirement for stock exchanges. In the U.S. (unlike many countries), firms are legally allowed to “go public” without listing on an exchange. Nevertheless, many U.S. firms that are going public plan to list immediately on a specific exchange, so listing requirements can influence the IPO allocation process. The New York Stock Exchange, the American Stock Exchange, and NASDAQ all require listing firms to have a certain minimum number of investors each owning at least one round lot. Since a round lot is a relatively small allocation, compared to the average holdings of an institutional investor, it would be relatively cheap to find some uninformed investors (perhaps some politicians, athletes, or other celebrities) and allocate each of them 100 shares simply to round out the investor lists.

To look for evidence of this, one could examine firms that are planning to list on an exchange and are at or close to the minimum investor number. If the allocation figures for these firms show a cluster of allocations at the minimum level, with perhaps a large gap between those minimum allocations and the allocations to other investors, this would indicate that the small investors are earning excess returns, and that the level of underpricing has been driven up by the listing requirement.

8. Conclusion

This paper explores the book-building process, which is increasingly being used throughout the world to distribute IPOs. By endogenizing the information acquisition process, we can examine how the underwriter selects the investors that participate in the offering and how the price and allocation schemes are determined. We demonstrate the tradeoff that the underwriter faces when increasing the size of the investor pool. If more investors are invited to participate, the issue can be priced more accurately. However, increased participation increases underpricing.

When information is costless, the optimal number of participating investors is infinite and underpricing approaches zero. When information is costly, the level of underpricing is determined by the desire for information. For firms with the most to gain from accurate pricing, more investors will be invited to participate in the offering and underpricing will be greater.

In some cases, the expected level of underpricing exactly compensates for the costs of acquiring information, leaving investors indifferent about whether or not to participate in IPOs. In other cases, however, investors realize excess returns from participating in new issues, and thus participation must be rationed. This will occur pointed out in Sherman (2000)—that hybrids increase underpricing by limiting the ability of the underwriter to link participation in one IPO to allocations in future IPOs. The increasing popularity of the hybrid method is documented in Sherman (2001).

13A firm that wants to list on the NYSE must have at least 2000 round lot holders (or a lower number of investors plus certain minimum trading volume levels, but trading volume alternatives presumably would not be available for an IPO that wanted to list immediately). Nasdaq requires at least 400 round lot holders for a National Market listing and 300 round lot holders for a SmallCap Market listing. The American Stock Exchange also requires a minimum of 400 round lot holders.
when the investment bank wants a large number of investors to participate in the offering, either because it wants better information or for regulatory or corporate governance reasons.

Empirical evidence in James (1997) suggests that underwriters form investor groups that regularly participate in the underwriters’ IPOs. These investor groups are consistent with the sets of participants described in our model. In addition, anecdotal evidence suggests that underwriters are rationing not only shares in select hot issues but also entrance into the investor groups, implying that at least part of the underpricing can be viewed as rents associated with buying IPOs. While we explain why these rents can arise in competitive markets, additional empirical work is needed to gauge the importance of the information and incentive issues raised in this paper.

Our analysis provides potentially testable empirical implications on the determinants of IPO underpricing. In particular, the model suggests that firms with a high need for price accuracy are likely to be the most underpriced. These firms might include riskier companies, smaller firms with shares that are likely to be more thinly traded, and firms that expect to have significant capital needs in the future. In addition, some firms’ operations might be particularly vulnerable to aftermarket price volatility, which could cause them to lose clients or to find it more difficult to attract top employees. These firms will be willing to pay for greater price discovery in the IPO process, since this is likely to lead to less volatility in the aftermarket.

Studying the book-building method in detail is important because of the growing international trend towards its use. However, this method is often unpopular with the public. There is an appearance that the system is less fair because it leaves out small investors. As a result, many countries that are beginning to allow book building are retaining or adding restrictions that force part of the shares in each IPO to be allocated to small investors. We show in this paper that these constraints increase the level of underpricing, thereby increasing the cost of going public.

Appendix A. Information reporting or truth-telling constraints

The full set of six truth-telling or information reporting constraints are given below, expanding Eq. (2). These conditions are discussed in Section 3.1.

\[ R(g, g) \geq R(g, u): \]
\[ P'(g, 0)\{(s^d - s_g)(q_{g,d,1}) - (s^d - s_u)(qu)\} + (1 - P'(g, 0))\{(s^d - s_g)(q^*_{g,d} - q_{u,d})\} \geq 0. \]
(A.1)

\[ R(u, u) \geq R(u, b): \]
\[ \frac{\pi(1 - \pi_t)}{2(1 - \pi_t)} \{1 - P'(b, 0))(s^b - s_b)(q_{u,b} - q^*_{b,b}) + (1 - P'(g, 0))\{(s^d - s_g)q_{u,g})\} + P'(u, 0)\{(s^u - s_u))q_u - (s^u - s_b)q_{b,b,1}\} \geq 0. \]
(A.2)

14This issue is considered in detail in Subrahmanyam and Titman (2001).
\[ R(g, g) \geq R(g, b) : \]
\[ P'(g, 0) \{(s^g - s_g)q_{g,g,1} - (s^g - s_b)q_{b,g,1}\} + (1 - P'(g, 0))\{(s^g - s_g)q_{g,g}^*\} \geq 0. \quad (A.3) \]

\[ R(u, u) \geq R(u, g) : \]
\[ \frac{\pi(1 - \pi_i)}{2(1 - \pi_i^2)} \{1 - P'(g, 0))\{(s^u - s_g)(q_{u,g} - q_{g,g}^*) + (1 - P'(b, 0))\{(s^b - s_b)q_{u,b}\} \]
\[ + P'(u, 0)\{(s^u - s_a)q^u - (s^u - s_g)q_{g,g,1}\} \geq 0. \quad (A.4) \]

\[ R(b, b) \geq R(b, u) : \]
\[ P'(b, 0)\{(s^b - s_b)q_{b,b,1} - (s^b - s_u)q_{u,b}\} + (1 - P'(b, 0))\{(s^b - s_b)q_{b,b}^* - q_{u,b}\} \geq 0. \quad (A.5) \]

\[ R(b, b) \geq R(b, g) : \]
\[ P'(b, 0)\{(s^b - s_b)q_{b,b,1} - (s^b - s_g)q_{g,g,1}\} + (1 - P'(b, 0))\{(s^b - s_b)q_{b,b}^* - q_{b,b}\} \geq 0. \quad (A.6) \]

Three of these conditions, Eqs. (A.4)–(A.6), limit the amount of underpricing in the model, since they all involve reporting better information than has been received—saying good if bad or uncertain, or saying uncertain if bad. Falsely reporting good information can raise the price of the issue but will also bring a larger allocation. If underpricing is excessive, investors will find it worthwhile to raise the price per share by reporting a “better” signal, since the better report would allow the investor to receive more of the still-underpriced shares.

The other three constraints, Eqs. (A.1)–(A.3), all involve underreporting—falsely reporting bad information or withholding good information. To prevent this, there must be at least a minimum level of underpricing, and allocations must be larger when better information is reported. In practice, the two constraints most likely to bind are (A.1) and (A.4). Eq. (A.1) provides conditions that keep investors from withholding good information by reporting \( u \) or “no information” when they receive the signal \( g \). Because it plays a major role in the solution of this model, we will explain the terms.

\( R(g, g) \geq R(g, u) \): An investor who receives signal \( g \) but reports \( u \) will receive \( q_u \) underpriced shares (worth \( s^g \) but priced at \( s_u \)) if, with probability \( P'(g, 0) \), no other investor receives a good signal. If, instead of lying, the investor had reported \( g \), the shares would have been less underpriced (\( s^g - s_g \) rather than \( s^g - s_u \)), but the investor would have received \( q_{g,g,1} \) shares rather than \( q_u \). If, with probability \( 1 - P'(g, 0) \), at least one other investor reports signal \( g \), then the price will be the same whether or not the first investor lies, but the first investor’s allocation of shares will on average be \( q_{u,g} \) if reporting \( u \) but \( q_{g,g}^* \) if reporting \( g \). Thus, to induce the investor to truthfully report \( g \), the advantage of possible greater underpricing if the investor lies must be offset by a lower expected allocation.
Appendix B. A numerical example

The equilibrium pricing and allocation strategy described in Proposition 1 and the propositions stated in Section 4 can be illustrated with a relatively simple numerical example. The example assumes the following values: $p = p_i = 0.5; c = 1.0; s^d = 1.0; s^g = 1.5; s^u = 0.5; \text{and } N = 100$. The underwriter in this example must choose the number of investors participating in the offer and come up with a pricing and allocation schedule that induces the investors to acquire information and reveal it truthfully.\(^{15}\)

B.1. Example with $H = 2$

First consider the case with only two investors. Given a good signal, shares are worth 1.5 each. If only one investor reports a good signal, that investor receives all 100 shares. If both investors report good signals, each receives 50 shares. Given that one investor receives a good signal, the chance that the other investor also receives a good signal is $p_i = 50\%$. Thus, the expected allocation for an investor who receives and reports a good signal is $0.5 \times 100 + 0.5 \times 50 = 75$. The unconditional probability that an investor will receive a good signal is $p_i p = 0.125$. The expected profit per share is $1.5 - s_g$.

To give investors the incentive to purchase signals, the shares must be underpriced sufficiently in state $g$ to compensate for the cost of the signals. Thus, the probability of receiving a good signal times the profit per share times the expected number of shares, both conditional on receiving a good signal, must be at least as big as 1.0, the cost of the signal, or $0.125(1.5 - s_g)75 \geq 1$, which simplifies to $s_g < 1.393$.\(^{16}\) If this constraint binds, underpricing in state $g$ will be 7.1%.

However, we also need to confirm that the investor has the incentive to accurately report his or her information, given the above pricing and allocation rule. The investor will not falsely report $b$ after observing $g$ or $u$ because it would lead to an allocation of zero (or, if others also report $b$, to an allocation of fairly priced state $b$ shares). Nor will the investor report $g$ after observing $u$ or $b$; or report $u$ after observing $b$, because such overreporting would lead to overpricing.

Finally, we must verify that the investor will not report $u$ after observing $g$. If other investors report $g$, then an investor who falsely reports $u$ will receive zero shares. If none of the other investors report $g$, then each investor receives $N/H = 50$ shares which are underpriced by 0.5 per share (since they are worth 1.5 but are priced at 1.0). Thus, the expected return to falsely reporting $u$ after observing $g$ is $0.5(1.5 - 1.0)50 = 12.5$. On the other hand, the investor who truthfully reports $g$ receives an expected allocation of 75 shares that are underpriced by $(1.5 - s_g)$ per share. Thus, to

\(^{15}\)In Section 3.1, there are many information reporting constraints that must be satisfied. For this example, however, we will only discuss binding conditions. It is easy to verify that the other conditions have been satisfied.

\(^{16}\)This is the information collection constraint, (3), which was given in Section 3.2. The constraint is greatly simplified because $s^h - s_b = s^d - s_u = q_{u,g} = 0$. 

induce the investor to accurately report his or her information, $s_g$ must satisfy $(1.5 - s_g)75 \geq 12.5$, or $s_g < 1.333$. Since this is lower than the 1.393 from the information purchase restriction, this constraint binds and $g$ shares are underpriced by 11.1%.

In this case, the expected amount that investors realize from underpricing exceeds the cost of acquiring information, implying that investors earn rents. The underwriter will thus have to restrict participation in the offer, due to excess demand. However, as we show below, the underwriter can both increase price accuracy and reduce the amount of underpricing by including more investors. Therefore $H = 2$ is not an equilibrium solution.

**B.2. Example with $H = 3$**

We now examine how the presence of a third participant affects the incentives of investors to purchase and reveal information. The method for solving this example is the same as that shown for $H = 2$. Inducing investors to purchase information when $H = 3$ requires that $s_g < 1.363$, while inducing investors to accurately report information requires that $s_g < 1.429$. Clearly, the information purchase constraint binds, so $s_g = 1.363$ and shares are underpriced by 9.1%.

Having more investors always increases pricing accuracy, since each additional investor generates an additional signal. With only two investors, the probability that an informative state is not discovered is $\pi(1 - \pi_i)^2 = 12.5\%$. With three investors, this probability drops to $\pi(1 - \pi_i)^3 = 6.25\%$. Hence, adding a third investor cuts the risk of not discovering the true state in half.

In this example, price accuracy increases and underpricing decreases as we go from two to three investors, showing that $H = 2$ is not optimal. The reason that underpricing decreases when we add a third investor is that, when $H = 2$, the truth-telling constraint is binding and this constraint is easier to satisfy when the number of investors increases. However, as the number of investors increases, the information collection constraint, which is more difficult to satisfy with higher $H$, becomes the binding constraint.

**B.3. Example with $H = 13$**

The key overreporting constraint in this model is Eq. (A.4), which guarantees that investors who observe $u$ will not falsely report $g$. When the issuer places a high value
on information and thus chooses a sufficiently high $H$, condition (A.4) cannot be satisfied unless (i) $q_{ug} > 0$, (ii) state $u$ shares are underpriced, and/or (iii) state $b$ shares are underpriced.

In our example, this occurs at $H = 13$. With only 12 investors, the price of state $g$ shares is 1.02, which means underpricing of 32.0% and a price for $g$ shares just barely above 1.0, the value of $u$ shares. Eq. (A.4) does not bind at $H = 12$, but it does bind at $H = 13$. With these parameters, the solution at $H = 13$ is to underprice $g$ shares by 32.7%, $u$ shares by 0.7%, and $b$ shares by 5.8%.20

When $H$ is sufficiently high, investors will earn excess returns. With $H = 13$, investors earn an expected excess return of 0.0371 in addition to the information cost of 1.0. For $H < 12$ the expected return exactly equals the cost of acquiring information, so that investors are indifferent about whether or not to participate in the offer.

Appendix C. Proofs of propositions

Proof of Proposition 1. With three states ($g$, $u$, and $b$), there are eight possible underpricing combinations: I. no underpricing; II. $u$ only; III. $b$ only; IV. $u$ and $b$ but not $g$; V. $g$ only; VI. $g$ and $u$ but not $b$; VII. $g$ and $b$ but not $u$; and VIII. underpricing in all three states.

Given our assumptions, outcome I will not be chosen by the issuer. Outcome II is not feasible because constraint (3) could not be satisfied for any $q_u > 0$. Outcome III is not feasible: satisfying (3) would require $q_{hb} > q_{ub}$ (since $1 - \pi_1(1 - \pi_1) > \pi_1$); (A.3) would require $q_{b,b,1} = 0$, which implies $q_{b,b} > q_{b,b}$, and (A.2) would require $q_{ub} > q_{b,b}$, which violates (3) and/or (A.3). Similarly, Outcome IV is not feasible because (3) would require $q_{b,b} > q_{u,b}$ and (A.1) would require $q_u = 0$, violating (A.2).

Thus, the only feasible outcomes are V–VIII, all of which involve underpricing of $g$ shares. Outcome V is feasible because (A.2), (A.3), (A.5), and (A.6) will not bind ((A.2) requires $q_{b,b,1} = 0$). This leaves (3), (A.1) and (A.4): (3) requires $q_{g,g} > q_{u,g}$; (A.1) can be satisfied for sufficiently high $H$, $q_{g,g}$, and $s^g - s_g$; and (A.4) will bind for sufficiently high $H$, placing limits on $q_{u,g}$ and $s^g - s_g$. Note that $q_{u,g} = 0$ unless (A.4) binds.

For sufficiently low $H$ such that (A.4) does not bind, outcome V is optimal. Outcome VII is not optimal for a low $H$ because to satisfy (A.2) with $s^b > s_b$ requires either $q_{u,g} > 0$ or $q_{u,b} > q_{b,b}$, either of which would make it more difficult to satisfy (3), leading to higher underpricing for the same $H$ (a suboptimal outcome). Next, outcome V dominates outcomes VI and VIII for low $H$ because $s^u > s_u$ makes it more

20 If we allow “fractional” investors, there is an extremely narrow range for $H$, from 12.026 to 12.029, where underpricing of only $u$ and not $b$ shares is optimal. As one might guess, there is a limit to how high $H$ can go. In this example, $H = 25$ is still feasible (with 100% underpricing of $b$ shares), but at $H = 26$, there is no set of non-negative prices that satisfies all of the constraints. This limit on $H$ is due to a feasibility constraint on how much the underwriter can offer.
expensive to satisfy (3) without any corresponding loosening of another binding restriction (because $H$ is low enough that (A.4) does not bind).

For a higher $H$, outcomes V, VI, VII, or VIII can be optimal, depending on the parameters. Outcome VI is more likely for low $\pi$, while outcome VII is more likely for high $\pi$. A higher $\pi$ means a higher probability of extreme values, either high or low, for the IPO shares.

Last, outcome VIII is always optimal for sufficiently high $H$: With outcome VII, it is not possible to lower $q_{u,g}$ to zero and still satisfy (A.2). Once the lower limit on $q_{u,g}$ has been reached with $s^d = s_u$, adding underpricing of $u$ shares will allow $q_{u,g}$ to be further lowered while satisfying (A.2), making the underpricing of $g$ shares more efficient. With outcome VI, the direct effect of $s^d > s_u$ is to make (3) harder to satisfy. However, this effect is small for high $H$, because $P(0; \sigma, 0)_{H}$ is small.

Underpricing $u$ makes it easier to satisfy (A.4), allowing $q_{u,g}$ to be lower, which will more than offset the direct effect on (3) (for sufficiently large $H$) and thus lower underpricing. However, if $s_u$ gets too low, (A.5) cannot be satisfied unless $s^b > s_b$.

**Proof of Proposition 2.** First, when Eq. (3) is binding, $\hat{c}(s^g - s_g) / \hat{c}H$ is positive. We examine the case where all underpricing occurs in state $g$, but the case where $u$ and/or $b$ shares are also underpriced is a straightforward extension. If Eq. (3) is simplified and rearranged, with equilibrium terms substituted out, we get

$$M \equiv \frac{2cH}{\pi N[1 - (1 - \pi_1)H]} - (s^g - s_g) = 0. \tag{C.1}$$

We can see that $\hat{c}M / \hat{c}(s^g - s_g) = -1 < 0$; and

$$\frac{\hat{c}M}{\hat{c}H} = \frac{2c}{\pi N[1 - (1 - \pi_1)H]} - \frac{2cH\pi_1(1 - \pi_1)H}{\pi N[1 - (1 - \pi_1)H]^2} \cdot$$

$$= \frac{2c}{\pi N[1 - (1 - \pi_1)H]} \left(1 - \frac{H\pi_1(1 - \pi_1)H}{1 - (1 - \pi_1)H}\right). \tag{C.2}$$

After rearranging the last term, we can see that this is positive if $1 > (1 - \pi_1H)(1 - \pi_1)H$. This condition is satisfied for $H = 1$, and the right-hand side decreases as $H$ increases. Therefore, when (3) is binding,

$$\hat{c}(s^g - s_g) / \hat{c}H = -\frac{\hat{c}M / \hat{c}H}{\hat{c}M / \hat{c}(s^g - s_g)} > 0. \tag{C.3}$$

Next, when (A.1) is binding, $\hat{c}(s^g - s_g) / \hat{c}H$ is negative. The proof of this is similar to the proof shown above for the case where (3) is binding, and this result is shown for a similar environment in Benveniste and Wilhelm.

Eqs. (A.1) and (3) both give minimum levels for underpricing of $g$ issues, $s^g - s_g$. These partial derivative results show that (A.1) is more likely to bind for a lower $H$ (since the minimum underpricing that it requires is highest when $H$ is low), while (3) is more likely to bind for a high $H$. Last, the underwriter will always choose an $H$
such that (3) rather than (A.1) is the binding constraint, since the higher $H$ leads to both less underpricing and greater price accuracy.

**Proof of Proposition 3.** To satisfy the information collection constraints, each investor’s expected return to participating must be better than the best alternative. In equilibrium, the best alternative would be to keep the cost of the signal, pretend to evaluate, and report “$u$” or uncertain. In this case, the investor’s expected return is equal to the cost of the signal, $c$, plus the expected excess return to “blindly” reporting $u$, $R(u, 0)$. This second term is

$$R(u, 0) = \frac{(\pi/2)(1 - P'(g, 0))}{(1 - P'(b, 0))}\{(s^g - s_g) q_{u,g}\} + \frac{(\pi/2)(1 - P'(b, 0))}{(1 - P'(b, 0))}\{(s^b - s_b) q_{u,b}\} + P'(u, 0)\{(s^u - s_u) q_u\}. \quad (C.4)$$

When (A.4) does not bind, this term is zero, since $q_{u,g} = 0$, $s^b = s_b$, and $s^u = s_u$. However, if (A.4) binds, then at least one of the following will be true: $q_{u,g} > 0$, $s^b > s_b$, or $s^u > s_u$. In this case, $R(u, 0)$ is strictly positive, and thus the expected return to investors must be strictly greater than $c$. □

**Proofs of Propositions 4–6.** We will examine the case in which the issuer chooses a sufficiently low $H$ that (A.4) does not bind. It is straightforward to extend these results to the other cases. Because the underwriter plans to sell $N$ shares regardless of the state, $q_{u,g}$ and $q_{u,b}$ are functions of $q_{g,g}$ and $q_{b,b}$. Also, $q_{u,g} = 0$ when (A.4) does not bind. From (A.2), $q_{b,b,1} = 0$. Last, using the facts that $P(g, 0) = P(b, 0)$, $2s^u = s^g + s^b$, $s_g = s^u$, and $s_b = s^b$, the first-order condition for $H$ reduces to $s^g - s_g = f'(P(b, 0))$. Thus, we have a system of two equations, (3) and the first-order condition for $H$, and two remaining unknowns, $s_g$ and $H$. After substitutions, these equations are as follows:

$$2cH \pi N[1 - (1 - \pi)^H] - (s^g - s_g) = 0 \quad (C.5)$$

and

$$f'(\pi (1 - \pi)^H) - (s^g - s_g) = 0. \quad (C.6)$$

Since Eqs. (C.5) and (C.6) are both differentiable and the determinant of the Jacobian is nonzero (as we will show), the Implicit Function Theorem can be used as long as the parameters are such that a solution exists. Thus, we can use Cramer’s Rule to obtain the comparative statics for this two-equation system. The determinant of the Jacobian is clearly negative and thus nonzero, as required by the Implicit Function Theorem. $|Jx, y|$ is the determinant of the matrix where the column of partial derivatives for $x$ is replaced with the column of partial derivatives for $y$ (where $x$ is the endogenous variable and $y$ is an exogenous variable).

To obtain get the partial derivatives, we use Cramer’s Rule: $\partial x/\partial y = -|Jx, y|/|J|$. The minus sign arises because we have a system of the form $Ax + b = 0$.
the usual $Ax = b$ (see Novshek, 1993, p. 173). The comparative statics are fairly straightforward, except for the “mixed” cases discussed in Section 5. Details are available from the authors. □

Proof of Proposition 7. First, $H$ will have to increase under either of these restrictions. Under limit I, $H$ must equal $H^* + 1 > H'$ so that we can still have $q_{b,h,1} = 0$. Under limit II, we again need to be able to set $q_{b,h,1} = 0$ to satisfy Eq. (A.2), so $H = N/q_{\text{max}} + 1 > H'$ (recall that $H'$ is the unconstrained optimal $H$).

Second, the underwriter will have to allocate some shares in $g$ issues to investors that report $u$ (i.e., $q_{u,g,h} > 0$ for at least some $h$) in order to satisfy the participation limits. However, the underwriter can adjust allocations and prices based on $h$ to satisfy (A.1)–(A.6) without allocating underpriced shares to investors that report $u$. The guideline will be that $s_{g,h} < q^d$ (underpricing) only for $h = H^* + 1$ or $h > N/q_{\text{max}}$, in which case $q_{u,g,h} = 0$.

The first-order condition for $H$ will no longer bind, since we no longer have an interior optimum. Eq. (3) will once again determine underpricing. We show in the proof of Proposition 2 that underpricing must increase when $H$ increases, as long as (3) is binding. Therefore, since $H$ has increased, the equilibrium underpricing must increase also. In extreme cases, the underwriter will be forced to allocate some underpriced shares in good issues to investors that report $u$, and the overall level of underpricing will have to increase even more. □

Proof of Proposition 8. Under either type of investor restriction, adding one uninformed investor to the syndicate will allow the underwriter to lower the number of informed investors (to $H^*$ under limit I and to $N/q_{\text{max}}$ under limit II), while still allowing the underwriter to set $q_{b,h,1} = 0$ and sell all $N$ shares. There is essentially no cost to having one uninformed investor, since that investor will only be allocated fairly priced (state $u$ or $b$) shares and will not receive the underpriced state $g$ shares. Since $H$ was above the optimal level before, costlessly lowering it toward the unconstrained optimum clearly increases the utility of the underwriter.

However, if more than one informed investor is replaced by an uninformed investor, it will become necessary to allocate some underpriced shares to an uninformed investor. Under restriction I, the number of informed investors that would be replaced by uninformed investors will depend primarily on the size of $\varepsilon$. If $\varepsilon$ is very small, the cost of allocating only $\varepsilon$ underpriced shares to each uninformed investor is relatively low. Thus, it might be worth adding uninformed investors in order to reduce the number of “extra” informed investors.

Under restriction II, there is no advantage to adding more than one uninformed investor, because informed investors are already receiving the maximum possible number of underpriced shares, $q_{\text{max}}$. If informed investor allocations do not change, then the amount of required underpricing will not change, as can be seen from Eq. (3) with $q_{g,d}$ replaced by $q_{\text{max}}$. By replacing informed with uninformed investors, the underwriter loses the additional information that would have been gained from the informed investors while getting nothing in return. □
References


