Do Fama-French Factors Proxy for Innovations in Predictive Variables?

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Definition: Innovations to predictors

The innovation to predictor $z_{kt}$ is the unexpected component $u^K_t$ of the variable.

$$z_{kt} = E \left[ z_{kt}\mid \Omega_{t-1} \right] + u^K_t$$

It is also referred to as the unexpected shock to $z_k$.

- According to the ICAPM, only the unexpected component of the state variable should command a risk premium.

- 4 Predictive variables:

RF is a short term T-bill yield, TERM is a term spread, DIV is the aggregate dividend yield, and DEF is the default spread.
Objectives

1) HML and SMB proxy for innovations in state variables that predict the excess market return and the yield curve.
2) Model in which factors are both excess market return and innovations in those 4 predictors explains the cross section of average returns better than the FF model.
3) When innovations in the predictors are present in the model, loadings on HML and SMB lose their explanatory power for the cross section of returns.
According to ICAPM, if investment opportunities change over time, then assets’ exposures to these changes are important determinants of average returns in addition to the market beta.

Assume the following general model for the unconditional expected excess returns $ER_i$ on assets

$$ER_i = \gamma_M \beta_{i,M} + \sum (\gamma_{uK}) \beta_{i,uK}, \forall i$$

Here $\gamma_M$ is the market risk premium, and $\gamma_{uK}$ is the price of risk for innovations in state variable $K$.

The betas are the slope coefficients from the return-generating process

$$R_{i,t} = \alpha_i + \beta_{i,M} R_{M,t} + \sum (\beta_{i,uK}) u^K_t + \epsilon_{i,t}, \forall i$$
Estimating the innovation terms

- We need to specify a process for time-series dynamics of predictors to extract innovations. VAR(1) is the candidate: \( z_t = Az_{t-1} + u_t \)

\[
\begin{pmatrix}
R_{M,t} \\
Div_t \\
TERM_t \\
DEF_t \\
RF_t \\
R_{HML,t} \\
R_{SMB,t}
\end{pmatrix} =
\begin{pmatrix}
a_{11} & \cdots & a_{17} \\
a_{21} & \cdots & a_{27} \\
a_{31} & \cdots & a_{37} \\
a_{41} & \cdots & a_{47} \\
a_{51} & \cdots & a_{57} \\
a_{61} & \cdots & a_{67} \\
a_{71} & \cdots & a_{77}
\end{pmatrix}
\begin{pmatrix}
R_{M,t-1} \\
Div_{t-1} \\
TERM_{t-1} \\
DEF_{t-1} \\
RF_{t-1} \\
R_{HML,t-1} \\
R_{SMB,t-1}
\end{pmatrix} +
\begin{pmatrix}
\hat{u}_t^R \\
\hat{u}_t^DIV \\
\hat{u}_t^{TERM} \\
\hat{u}_t^{DEF} \\
\hat{u}_t^{RF} \\
\hat{u}_t^{HML} \\
\hat{u}_t^{SMB}
\end{pmatrix}
\]

This VAR represents a joint specification of the dynamics of All candidate state variables within the ICAPM.
Testing the ICAPM specification

Since not all factors represent portfolio returns Fama-McBeth cross-sectional method is appropriate. First step:

\[ R_{i,t} = \alpha_i + \beta_{i,M} R_{M,t} + \beta_{i, u^{DIV}} \hat{u}_{t}^{DIV} + \beta_{i, u^{TERM}} \hat{u}_{t}^{TERM} + \beta_{i, u^{DEF}} \hat{u}_{t}^{DEF} + \beta_{i, u^{RF}} \hat{u}_{t}^{RF} + \beta_{i, u^{HML}} \hat{u}_{t}^{HML} + \beta_{i, u^{SMB}} \hat{u}_{t}^{SMB} + \epsilon_{i,t}, \quad \forall \; i \]

Second step: we specify the cross-sectional relation:

\[ R_{i,t} = \gamma_o + \gamma_{M} \hat{\beta}_{i,M} + \gamma_{u^{DIV}} \hat{\beta}_{i, u^{DIV}} + \gamma_{u^{TERM}} \hat{\beta}_{i, u^{TERM}} + \gamma_{u^{DEF}} \hat{\beta}_{i, u^{DEF}} + \gamma_{u^{RF}} \hat{\beta}_{i, u^{RF}} + \gamma_{u^{HML}} \hat{\beta}_{i, u^{HML}} + \gamma_{u^{SMB}} \hat{\beta}_{i, u^{SMB}} + \epsilon_{i,t}, \quad \forall \; t \]

Relation between innovations and FF factors: Table

Regression: \( \hat{u}_t = c_0 + c_1 R_{M,t} + c_2 R_{HML,t} + c_3 R_{SMB,t} + \varepsilon_t \)

<table>
<thead>
<tr>
<th>Dep. Variable</th>
<th>( c_0 )</th>
<th>( c_1 )</th>
<th>( c_2 )</th>
<th>( c_3 )</th>
<th>Adj. ( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{u}_t^{DIV} )</td>
<td>0.00</td>
<td>-0.08</td>
<td>-0.30</td>
<td>-0.01</td>
<td>3.00</td>
</tr>
<tr>
<td></td>
<td>0.85</td>
<td>-0.70</td>
<td>-2.43</td>
<td>-0.09</td>
<td></td>
</tr>
<tr>
<td>( \hat{u}_t^{TERM} )</td>
<td>-0.00</td>
<td>0.06</td>
<td>0.24</td>
<td>0.03</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>-0.56</td>
<td>0.75</td>
<td>2.30</td>
<td>0.59</td>
<td></td>
</tr>
<tr>
<td>( \hat{u}_t^{DEF} )</td>
<td>-0.00</td>
<td>0.07</td>
<td>0.17</td>
<td>-0.12</td>
<td>2.00</td>
</tr>
<tr>
<td></td>
<td>-0.38</td>
<td>1.11</td>
<td>2.10</td>
<td>-1.92</td>
<td></td>
</tr>
<tr>
<td>( \hat{u}_t^{RF} )</td>
<td>0.00</td>
<td>-0.04</td>
<td>-0.13</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>0.36</td>
<td>-0.51</td>
<td>-1.36</td>
<td>0.14</td>
<td></td>
</tr>
</tbody>
</table>
Relation between innovations and FF factors

Discussion

- \( corr \left( R_{HML}, \hat{u}_{HML} \right) = 0.90, \ corr \left( R_{SMB}, \hat{u}_{SMB} \right) = 0.92. \) Returns on the HML and SMB portfolios are good proxies for the innovations. Or, Petkova has a bad econometric model for HML and SMB that doesn’t capture dynamics of those returns. As a result, dynamics of FF factors is in the errors, that is why high correlation.

- If you work with 4 predictors only - same intuition and result.

- Significant corr with SMB: innovations for DEF (—). Bigger stocks are able to track long-run trends in the business cycle better than the small stocks.
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Relation between innovations and FF factors

Discussion

- $\text{corr}(\ HML, \ TERM \ ) > 0$ Values of the term spread signal that expected market returns are low during expansions and high during recessions. The TERM spread very closely tracks the short-term fluctuation in the business cycle. Positive shocks to the TERM premium are associated with bad times (everyone is averse to long term investment due to high risk during recession, therefore TERM spread is high) and negative - with good times.

- Petkova and Zhang (2004) documented that value stocks are riskier than growth stocks in bad times and less risky during good times. $\Rightarrow \text{corr}(\ HML, \ TERM \ ) > 0$. Therefore, negative TFP is associated with higher HML and higher TERM spread, and vice versa.

- $\text{corr}(\ HML, \ DEF) > 0$: HML is a measure of distress risk.
Relation between innovations and FF factors

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- \( \text{corr}(HML, DEF) > 0 \) : HML is a measure of distress risk.
Are HML and SMB significant risk factors in the presence of the innovations?

<table>
<thead>
<tr>
<th>( \gamma_0 )</th>
<th>( \gamma_M )</th>
<th>( \gamma_{u,\text{DIV}} )</th>
<th>( \gamma_{u,\text{TERM}} )</th>
<th>( \gamma_{u,\text{DEF}} )</th>
<th>( \gamma_{u,\text{RF}} )</th>
<th>( \gamma_{u,\text{HML}} )</th>
<th>( \gamma_{u,\text{SMB}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.11</td>
<td>-0.57</td>
<td>-0.83</td>
<td>3.87</td>
<td>0.37</td>
<td>-2.90</td>
<td>0.42</td>
<td>0.41</td>
</tr>
<tr>
<td>3.29</td>
<td>-1.45</td>
<td>-0.94</td>
<td>3.53</td>
<td>0.42</td>
<td>-3.33</td>
<td>1.62</td>
<td>1.75</td>
</tr>
<tr>
<td>2.36</td>
<td>-1.10</td>
<td>-0.69</td>
<td>2.56</td>
<td>0.31</td>
<td>-2.44</td>
<td>1.40</td>
<td>1.56</td>
</tr>
</tbody>
</table>

Hypothesis that shocks in 4 predictors span the information contained in the FF factors cannot be rejected. \( R^2 \) is 77%.
### 2 alternative models: Table

#### Panel A: The Fama–French Three-Factor Model

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_0$</th>
<th>$\gamma_M$</th>
<th>$\gamma_HML$</th>
<th>$\gamma_SMB$</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>1.15</td>
<td>-0.65</td>
<td>0.44</td>
<td>0.16</td>
<td>71.00</td>
</tr>
<tr>
<td>FM $t$-stat</td>
<td>3.30</td>
<td>-1.60</td>
<td>3.09</td>
<td>1.04</td>
<td></td>
</tr>
<tr>
<td>SH $t$-stat</td>
<td>3.19</td>
<td>-1.55</td>
<td>3.07</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

#### Panel B: The Model with $R_M$ and Innovations in $DIV$, $TERM$, $DEF$, and $RF$

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_0$</th>
<th>$\gamma_M$</th>
<th>$\gamma_{\hat{\alpha}}^{DIV}$</th>
<th>$\gamma_{\hat{\alpha}}^{TERM}$</th>
<th>$\gamma_{\hat{\alpha}}^{DEF}$</th>
<th>$\gamma_{\hat{\alpha}}^{RF}$</th>
<th>Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimate</td>
<td>0.64</td>
<td>-0.07</td>
<td>-1.39</td>
<td>4.89</td>
<td>-0.54</td>
<td>-3.22</td>
<td>77.00</td>
</tr>
<tr>
<td>FM $t$-stat</td>
<td>1.74</td>
<td>-0.16</td>
<td>-1.56</td>
<td>4.44</td>
<td>-0.58</td>
<td>-3.79</td>
<td></td>
</tr>
<tr>
<td>SH $t$-stat</td>
<td>1.08</td>
<td>-0.11</td>
<td>-0.99</td>
<td>2.79</td>
<td>-0.37</td>
<td>-2.40</td>
<td></td>
</tr>
</tbody>
</table>
Even though $\beta_M$ is not priced in the cross section of average returns, presence of this variable tends to significantly increase the cross-sectional $R^2$.

Model B can’t be rejected using Shanken’s t-statistics while the FF model is rejected over the period $(\gamma_{0}^{FF} > 0)$ and significant.

Asset covariances with a term spread and a T-bill surprise factor are important in the cross-section of average portfolio returns.

HML proxies for a term spread surprise factor in returns while SMB proxies for a default spread surprise factor.
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