2. GARCH Models

The Mean Equation

\[ E_t r_{t+1} = \mu + \gamma \sigma_t^2 \]
\[ V_t r_{t+1} = \sigma_t^2 \]

\[ r_{t+1} = \mu + \gamma \sigma_t^2 + \eta_{t+1} \]

\[ r_{t+1} \equiv \ln(\text{CRSP value weighted index return}) - \ln(\text{T-bill rate}) \]
\[ \eta_{t+1} (r_{t+1}) \text{ has conditional variance } \sigma_t^2 \]

For \( \epsilon_{t+1} \) i.i.d. \( \sim \phi(0, 1) \)

\[ \eta_{t+1} = \sigma_t \epsilon_{t+1} \]

\[ \eta_{t+1} \sim \phi(0, \sigma_t^2) \]

\[ r_{t+1} = \mu + \gamma \sigma_t^2 + \sigma_t \epsilon_{t+1} \]
2. GARCH Models

The Mean Equation

\[ E_t r_{t+1} = \mu + \gamma \sigma_t^2 \]
\[ V_t r_{t+1} = \sigma_t^2 \]

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\[ \eta_{t+1} (r_{t+1}) \text{ has conditional variance } \sigma_t^2 \]

For \( \epsilon_{t+1} \text{ i.i.d. } \sim \phi(0,1) \)
\[ \eta_{t+1} = \sigma_t \epsilon_{t+1} \]
\[ \eta_{t+1} \sim \phi(0, \sigma_t^2) \]

\[ r_{t+1} = \mu + \gamma \sigma_t^2 + \sigma_t \epsilon_{t+1} \]
The Variance Equation

**Canonical GARCH**
Bollerslev (1986)

\[ \sigma_t^2 = \omega + \alpha \sigma_{t-1}^2 \epsilon_t^2 + \beta \sigma_{t-1}^2 \]

- large shocks → very large \( \sigma_t^2 \)
- symmetry

**Absolute Value GARCH**

\[ \sigma_t = \omega + \alpha \sigma_{t-1} |\epsilon_t| + \beta \sigma_{t-1} \]

- symmetry
Modelling Asymmetry

\[\sigma_t = \omega + \alpha \sigma_{t-1} |\epsilon_t| + \beta \sigma_{t-1}\]

\[\sigma_t = \omega + \alpha \sigma_{t-1} |\epsilon_t - b| + \beta \sigma_{t-1}\]

\[\sigma_t = \omega + \alpha \sigma_{t-1} \left[ |\epsilon_t| - c\epsilon_t \right] + \beta \sigma_{t-1}\]

\[\sigma_t = \omega + \alpha \sigma_{t-1} \left[ |\epsilon_t - b| - c(\epsilon_t - b) \right] + \beta \sigma_{t-1}\]

\[f(\epsilon_t) = |\epsilon_t - b| - c(\epsilon_t - b)\]

for positivity we need \(|c| \leq 1\)
The News Impact Curve
Pagan and Schwert (1990), Engle and Ng (1991)

How do shocks affect conditional volatility?

Plot surprise in conditional volatility against shock:

\[ (\sigma_t - E_{t-1}\sigma_t) \quad \text{vs.} \quad \epsilon_t \]

for \( \epsilon_t \) i.i.d. \( \phi(0,1) \) \( E_{t-1}|\epsilon_t| = \text{constant} \)

\[
\begin{align*}
\sigma_t &= \omega + \alpha\sigma_{t-1}|\epsilon_t| + \beta\sigma_{t-1} \\
E_{t-1}\sigma_t &= \omega + \alpha\sigma_{t-1}E_{t-1}|\epsilon_t| + \beta\sigma_{t-1} \\
(\sigma_t - E_{t-1}\sigma_t) &= \alpha\sigma_{t-1}|\epsilon_t| - \alpha\sigma_{t-1}E_{t-1}|\epsilon_t| \\
(\sigma_t - E_{t-1}\sigma_t) &\propto |\epsilon_t|
\end{align*}
\]
3. A Family of Variance Models
Nest existing GARCH models:
1. Unify GARCH literature
2. Test for “best” model using nested tests

Two-step procedure:
1. Box-Cox transformation of $\sigma_t$
2. Power transformation of $f(\epsilon_t)$

$$\frac{\sigma_t^\lambda - 1}{\lambda} = \omega' + \alpha \sigma_{t-1}^{\lambda} f' \nu(\epsilon_t) + \beta \frac{\sigma_{t-1}^\lambda - 1}{\lambda}$$

$$f(\epsilon_t) = |\epsilon_t - b| - c(\epsilon_t - b)$$
4. Estimation
Full Maximum Likelihood Estimation for efficiency

\[ \epsilon_{t+1} \sim N(0, 1) \]

- quasi maximum likelihood White (1982)
- leptokurtic distributions don’t change parameters of interest
  - Bollerslev (1987): Student-\(t\)
Maximum Likelihood Estimation

- we observe $y_t$
- our model says that $y_t = f(\theta, x_t) + \epsilon_t$
- we take $x_t$ as given
- conditional on this model, the probability of observing $y_t$ is $L(\theta, x_t)$
- alternatively, conditional on the observations and the structure of the model, the probability that the observed data were generated by $\theta$ is also $L(\theta, x_t)$
- MLE tries to find $\theta$ by maximizing $\sum_t \ln L(\theta, x_t)$
Numerical Optimization

General Idea of Newton-Raphson Method

• want to maximize objective function $L(\theta)$
• true maximum is at $\theta$
• finding the maximum is equivalent to finding a zero gradient

$$L'(\theta_i) = L'\left(\theta + (\theta_i - \theta)\right) = L'(\theta + h)$$

• take a Taylor series approximation around the true maximum

$$L'(\theta + h) = L'(\theta) + hL''(\theta)$$

$$h = \frac{L'(\theta + h)}{L''(\theta)}$$

$$\theta = \theta_i - \frac{L'(\theta_i)}{L''(\theta)}$$
Recursive Nature of GARCH

\[ r_{t+1} = \mu + \gamma \sigma^2_t + \sigma_t \epsilon_{t+1} \]
\[ \sigma^2_t = \omega + \alpha \sigma^2_{t-1} \epsilon^2_t + \beta \sigma^2_{t-1} \]

- we observe \( r_{t+1} \)
- conditional on \( \sigma_t \) we can solve for \( \epsilon_{t+1} \)
  - need \( \sigma_0 \) to start everything up
  - unconditional expectation of \( \sigma_t \) makes a good starting point
- use \( \{ \epsilon_t \} \) to construct likelihood function

\[ L(\theta) = \sum_t \ln (\phi(\epsilon_t)) \]
The panels show the shifted and rotated absolute value function
\[ f(\epsilon_t) = |\epsilon_t - b| - c(\epsilon_t - b) \]
for various shifts and rotations, \( b \) and \( c \). The dashed line shows the absolute value function, \(|\epsilon_t|\), for comparison. The transformation \( f(\epsilon_t) \) controls the effect of shocks on the conditional volatility in the \textit{agarch} model
\[ \sigma_t = \omega + \alpha \sigma_{t-1} f(\epsilon_t) + \beta \sigma_{t-1}. \]

In the interest of brevity, I refer to this absolute value GARCH equation as the \textit{agarch}(1, 1) model.\(^1\)

The ‘news impact curve’ introduced by Pagan and Schwert (1990), and so christened by Engle and Ng (1993), is a useful tool in discussing asymmetry. The news impact curve relates revisions in conditional volatility to shocks. In the context of the absolute value GARCH model, it is convenient to investigate the impact of shocks on the conditional standard deviation. As figure 1a shows, the news impact curve of equation (4) is symmetric in \( \epsilon_t - \sigma_t \) space.

The \textit{agarch} model incorporates both of the main approaches previously

---

\(^1\)Engle and Ng (1993) named a variance equation that specifies the conditional variance as a function of a shifted parabola the \textit{agarch} model. This model has been more fully worked out by Sentana (1991) who refers to it as \textit{qgarch}. I hope that this duplication of acronyms does not lead to confusion.
Figure 2: The Transformation $f^\nu(\epsilon_t)$

The panels show the transformation $f^\nu(\epsilon_t)$ for

$$f(\epsilon_t) = |\epsilon_t - b| - c(\epsilon_t - b)$$

and different values of $\nu$, $b$ and $c$. The transformation $f^\nu(\epsilon_t)$ controls the impact of shocks on the transformed conditional volatility in the variance equation

$$\frac{\sigma_t^2 - 1}{\lambda} = \omega' + \alpha \sigma_{t-1}^\lambda f^\nu(\epsilon_t) + \beta \frac{\sigma_{t-1}^2 - 1}{\lambda}. \quad (7)$$

All conditional variance models should ensure that $\sigma_t^2$ takes real, positive values. In the case of Bollerslev’s (1986) GARCH model, this requirement implies that $\omega$, $\alpha$ and $\beta$ have to be positive. These conditions are sufficient but not always necessary for positive conditional variances in other models in the family.\(^2\)

Additionally, many models require that $f^\nu(\epsilon_t)$ is positive to ensure the positivity of $\sigma_t^2$. Positivity of $f^\nu(\epsilon_t)$ is guaranteed when $|c| \leq 1$, which ensures that neither arm of the rotated absolute value function crosses the abscissa. The parameter $b$, however, is unrestricted in size and sign. For all of the models in the family, there are restrictions on the magnitude of the parameters to ensure covariance

\(^2\)Slightly negative values of $\alpha_i$ and $\beta_j$ for higher-order lags don’t result in negative conditional volatility in any of these models. See Nelson and Cao (1992) for necessary restrictions on $\alpha_i$ and $\beta_j$ in the standard GARCH($p,q$) model with $p = \{0,1,2\}$. 
Table 1
Nested GARCH Models

<table>
<thead>
<tr>
<th>λ</th>
<th>ν</th>
<th>b</th>
<th>c</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>free Exponential GARCH (Nelson)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>free</td>
<td>Threshold GARCH (Zakoian)</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>≤1</td>
<td>Absolute Value GARCH (Taylor/Schwert)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>garch (Bollerslev)</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0</td>
<td>free</td>
<td>Nonlinear-Asymmetric GARCH (Engle, Ng)</td>
</tr>
<tr>
<td>free</td>
<td>λ</td>
<td>0</td>
<td>≤1</td>
<td>Asymmetric Power ARCH (Ding, Granger, Engle)</td>
</tr>
</tbody>
</table>

The first four columns list the restrictions one has to apply to the variance equation

\[
\frac{\sigma^2_t}{\lambda} = \omega + \alpha \sigma^2_{t-1} f(\epsilon_t) + \beta \left( \frac{\sigma^2_{t-1}}{\lambda} \right)
\]

in order to obtain the model named in the last column.

Stationarity of the process. All of the restrictions for positivity and stationarity are discussed in appendix A. Typically none of these restrictions are binding. Consequently, they are not an important factor in model choice and have been suppressed in table 1.

2.2.1 Exponential GARCH \((\lambda = 0, \nu = 1, \ b = 0)\)

Via L’Hospital’s rule one can show that the Box-Cox transformation converges to the natural logarithm as \(\lambda\) goes to zero: \(\lim_{\lambda \to 0} \left( \frac{\sigma^2_t}{\lambda} - 1 \right) = \ln \sigma_t\).

From L’Hospital’s rule it is clear that, for \(\nu = 1\), equation (7) converges to the exponential GARCH model introduced by Nelson (1991) as \(\lambda\) goes to zero. If \(b\) is set to zero, and the constant, unconditional mean of \(f(\epsilon_t)\) is subtracted from \(f(\epsilon_t)\) and added to the intercept, the variance equation becomes

\[
\ln \sigma^2_t = 2\omega'' + 2\alpha \left( |\epsilon_t| - E|\epsilon_t| - c|\epsilon_t| \right) + \beta \ln \sigma^2_{t-1},
\]

which is identical to Nelson’s E\(GARCH\). Since exponentiation also ensures positivity, E\(GARCH\) does not impose sign restrictions on \(\omega''\), \(\alpha\) and \(\beta\). More generally, equation (7) allows not only for the rotation of the news impact curve given in the standard formulation of Nelson’s E\(GARCH\), but also for the shift that was discussed in the previous section. This can be seen by substituting \((\epsilon_t - b)\) for \(\epsilon_t\) in equation (8).
### Table 2
Estimates of Variance Equation Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>$\lambda$ (SE)</th>
<th>$\nu$ (SE)</th>
<th>$\omega$ (SE)</th>
<th>$\alpha$ (SE)</th>
<th>$\beta$ (SE)</th>
<th>$b$ (SE)</th>
<th>$c$ (SE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EGARCH</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>TGARCH</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>AGARCH</td>
<td>1.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>GARCH</td>
<td>2.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>N-A GARCH</td>
<td>2.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>GJR-GARCH</td>
<td>2.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>NARCH</td>
<td>1.651</td>
<td>0.101</td>
<td>4.560</td>
<td>0.102</td>
<td>0.905</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>A-PARCH</td>
<td>1.380</td>
<td>0.112</td>
<td>18.990</td>
<td>0.088</td>
<td>0.916</td>
<td>0.000</td>
<td>0.334</td>
</tr>
<tr>
<td>A-PARCH</td>
<td>1.536</td>
<td>0.083</td>
<td>1.065</td>
<td>0.097</td>
<td>0.894</td>
<td>0.370</td>
<td>0.070</td>
</tr>
<tr>
<td>A-PARCH</td>
<td>1.131</td>
<td>0.249</td>
<td>7.594</td>
<td>0.070</td>
<td>0.918</td>
<td>0.394</td>
<td>0.050</td>
</tr>
</tbody>
</table>

The table presents parameter estimates for the family of models given by

\[
\begin{align*}
    \ln \sigma_t^2 &= \omega + \alpha \{ f(\epsilon_t) - E[f(\epsilon_t)] \} + \beta \ln \sigma_{t-1}^2 \\
    \sigma_t^2 &= \omega + \alpha \sigma_{t-1}^2 f(\epsilon_t) + \beta \sigma_{t-1}^2 \\
    \sigma_t^\lambda &= \omega + \alpha \sigma_{t-1}^\lambda f^\lambda(\epsilon_t) + \beta \sigma_{t-1}^\lambda \\
    r_{t+1} &= \mu + \gamma \sigma_t^2 + \sigma \epsilon_{t+1} \\
    \frac{\sigma_t^\lambda - 1}{\lambda} &= \omega + \alpha \sigma_{t-1}^\lambda f^\lambda(\epsilon_t) + \beta \frac{\sigma_{t-1}^\lambda - 1}{\lambda} \\
    f(\epsilon_t) &= \sqrt{\alpha^2 + (\epsilon_t - b)^2} - c(\epsilon_t - b),
\end{align*}
\]

where the variance equation coefficients, $\omega$, $\alpha$, and $\beta$, are normalized as shown in the table.

The parameter estimates were obtained by numerically maximizing the likelihood function for $\epsilon_t \sim N(0, 1)$. The daily log excess stock returns, $r_{t+1}$, include dividends. The stock returns are a combination of the Schwert (1990) returns and the value-weighted index returns from the CRSP tapes. The Fama/Bliss riskless interest rates were obtained from the CRSP tapes as well. The sample period, January 2, 1926–December 31, 1990, spans 17,486 observations. The parameter $a$ was fixed at 0.001. The numbers in parentheses are asymptotic standard errors computed from $A^{-1}BA^{-1}$, where $A$ is the Hessian matrix of the likelihood function with respect to the parameters and $B$ is the matrix of the outer products of the gradients.
The figure shows the news impact curves for five different GARCH models. The news impact curves were estimated from the daily log return on a value-weighted equity index in excess of the risk-free interest rate. The stock returns are a composite of the Schwert (1990) index returns and the CRSP index returns. The Fama/Bliss riskless rates were obtained from the CRSP bond files. The entire sample period, January 2, 1926 to December 31, 1990, spans 17,486 observations. The conditional standard deviations, $\sigma_t$, of excess returns, $r_{t+1}$, were estimated from the family of models described by

$$r_{t+1} = \mu + \gamma \sigma_t^2 + \sigma_t \epsilon_{t+1}$$

$$\frac{\sigma_t^2 - 1}{\lambda} = \omega + \alpha \sigma_{t-1}^2 f'(\epsilon_t) + \beta \frac{\sigma_{t-1}^2 - 1}{\lambda}$$

$$f(\epsilon_t) = \sqrt{a^2 + (\epsilon_t - b)^2} - c(\epsilon_t - b).$$

The appropriate restrictions on $\lambda$ and $\nu$ were enforced, but $b$ and $c$ were freely estimated. The maximum likelihood parameter estimates for each of the models are given in the bottom rows of the five panels in table 2. The responses shown are conditional on $\sigma_{t-1} = .01$, and were roughly converted to annual percentage terms with a scale factor of 1,600.

Although the likelihood ratio tests indicate that the models differ from each other, the tests don’t reveal how the models differ from each other. Figure 3 plots the news impact curves based on the parameter estimates for the five fully asymmetric models in the bottom row of each panel of table 2. All of the news impact curves in the figure assume that last period’s conditional volatility, $\sigma_{t-1}$, was 0.01, which is very close to the sample mean. The volatilities were roughly estimated, or at least is between one and two.
### Table 3
Likelihood Ratio Tests for Asymmetry in Volatility

<table>
<thead>
<tr>
<th>Maintained Hypothesis</th>
<th>$H_0$</th>
<th>$H_A$</th>
<th>$b = c = 0$</th>
<th>$b = 0$, $c$ free</th>
<th>$b$ and $c$ free</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c = 0$, $b$ free</td>
<td>303.622</td>
<td>304.542</td>
<td>311.667</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$b = 0$, $c$ free</td>
<td>8.045</td>
<td>7.125</td>
<td></td>
<td>(0.005)</td>
<td>(0.008)</td>
</tr>
<tr>
<td>$\lambda = 0$, $\nu = 1$</td>
<td>315.098</td>
<td>335.082</td>
<td>336.833</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\lambda = 1$, $\nu = 1$</td>
<td>306.023</td>
<td>230.990</td>
<td>306.114</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\lambda = 2$, $\nu = 2$</td>
<td>307.949</td>
<td>280.136</td>
<td>310.810</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\lambda = \nu$</td>
<td>309.179</td>
<td>275.798</td>
<td>310.654</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
<tr>
<td>$\lambda$, $\nu$ free</td>
<td>1.475</td>
<td>34.857</td>
<td></td>
<td>(0.224)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

The likelihood ratios test restrictions in the model

\[ r_{t+1} = \mu + \gamma \sigma_t^2 + \sigma_t \epsilon_{t+1} \]  
(1)

\[ \sigma_t^2 - 1 = \omega + \alpha \sigma_{t-1}^2 f^2(\epsilon_t) + \beta \left( \frac{\sigma_t^2}{\lambda} - 1 \right) \]  
(7)

\[ f(\epsilon_t) = \sqrt{a^2 + (\epsilon_t - b)^2 - c(\epsilon_t - b)}. \]  
(6)

The numbers in parentheses are asymptotic probability values. All parameter estimates were obtained by maximum likelihood estimation under the assumption that $\epsilon_t \sim N(0, 1)$, and a subset of the parameter estimates is given in table 2. The daily log excess stock returns, $r_{t+1}$, are measured as the log returns on the value-weighted index of NYSE stocks in excess of the log of the risk-free interest rate. The stock returns are a combination of the Schwert (1990) returns and the value-weighted index returns from the CRSP tapes. The riskless interest rates were obtained from the Fama/Bliss series, also on the CRSP tapes. The sample period, January 2, 1926–December 31, 1990, spans 17,486 observations.
Table 4
Likelihood Ratio Tests of Functional Form

<table>
<thead>
<tr>
<th></th>
<th>$H_0$</th>
<th>$H_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda = \nu$</td>
<td>$\lambda$, $\nu$ free</td>
</tr>
<tr>
<td>EGARCH</td>
<td>$\lambda = \nu$</td>
<td>60.758</td>
</tr>
<tr>
<td></td>
<td>$\lambda = 0$, $\nu = 1$</td>
<td>(0.000)</td>
</tr>
<tr>
<td>AGARCH</td>
<td>$\lambda = 1$, $\nu = 1$</td>
<td>75.089, 81.443</td>
</tr>
<tr>
<td>GARCH</td>
<td>$\lambda = 2$, $\nu = 2$</td>
<td>21.563, 27.918</td>
</tr>
<tr>
<td>NARCH</td>
<td>$\lambda = \nu$</td>
<td>6.355</td>
</tr>
</tbody>
</table>

The likelihood ratios test restrictions in the family of models given by

\[
\begin{align*}
\epsilon_{t+1} &= \mu + \gamma \sigma_t^2 + \sigma_t \epsilon_{t+1} \\
\frac{\lambda - 1}{\lambda} &= \omega + \alpha \sigma_{t-1}^2 f'(\epsilon_t) + \beta \frac{\sigma_{t-1}^2 - 1}{\lambda} \\
f(\epsilon_t) &= \sqrt{a^2 + (\epsilon_t - b)^2} - c(\epsilon_t - b),
\end{align*}
\]

where $\epsilon_{t+1}$ is the daily log return on a value-weighted equity index in excess of the risk-free interest rate. The stock returns are a composite of the Schwert (1990) index returns and the CRSP index returns. The Fama/Bliss riskless rates were obtained from the CRSP bond files. The parameters $b$ and $c$ are freely estimated in all models. The numbers in parentheses are asymptotic probability values. All parameter estimates were obtained by maximum likelihood estimation and are given in table 2. The sample runs from January 2, 1926 to December 31, 1990 and includes 17,486 observations.

both null hypotheses can easily be rejected. When $\lambda$ and $\nu$ are restricted to be equal to one another they are clearly not equal to either one or two, thereby rejecting the AGARCH and GARCH specifications, even in their asymmetric forms. The EGARCH model is not nested within the alternative hypothesis, and no test statistic is reported. The third column, however, tests all four null hypotheses against the general alternative that the data were generated by a model in which $\lambda$ and $\nu$ are different. Obviously, the second and third null hypotheses are also rejected by this even more general alternative. Additionally, the possibility that the data are generated by an exponential GARCH model is rejected in favor of the more general alternative. The $\chi^2$ test statistic of 60.758 is far above all standard critical values. Lastly, the test of the restriction that $\lambda = \nu$ barely falls below the $\chi^2$ one percent critical value of 6.635. In view of the very large sample size this rejection is rather weak, and for many purposes it is probably defensible to describe the daily excess returns with models in which $\lambda = \nu$ but the exponent