

**An integrated view of tests of rationality, market efficiency, and  
the short-run neutrality of monetary policy**

Andrew Abel and Frederic Mishkin  
(JME, 1983)

**Returns to contrarian investment strategies: Tests of naive  
expectations hypotheses**

Patricia Dechow and Richard Sloan  
(JFE, 1997)

## Abel and Mishkin (1983)

Two types of tests of rational expectations (or market efficiency)

- (1) Does the market's belief about, say, the dividend process equal the true process? (Cross-equation restrictions)
- (2) Can we forecast stock returns based on past information? (Fama-style regression test)

## Rational expectations

AM take 'rational expectations' to mean that the market's expectation of any variable equals the true expectation:

Let  $E_{t-1}^M[X_t] = X^e$ . Then RE implies  $X^e = E[X_t | \phi_{t-1}]$ .

For example, suppose

$$X_t = Z_{1,t-1} \alpha_1 + Z_{2,t-1} \alpha_2 + u_t, \quad \text{where } E[u_t | \phi_{t-1}] = 0.$$

In the following equation

$$X_t^e = Z_{1,t-1} \alpha_1^* + Z_{2,t-1} \alpha_2^*,$$

RE implies that  $\alpha_1^* = \alpha_1$ ,  $\alpha_2^* = \alpha_2$ .

## Rational expectations, cont.

Thinking about this differently:

$$X_t - X_t^e = Z_{1,t-1} (\alpha_1 - \alpha_1^*) + Z_{2,t-1} (\alpha_2 - \alpha_2^*) + u_t$$

With  $\alpha_1^* = \alpha_1$ ,  $\alpha_2^* = \alpha_2$ , it is clear that rational expectations implies that the market's forecast error must be unpredictable. In other words,

$$E_{t-1}[X_t - X_t^e] = 0 \quad \rightarrow \quad \text{forecast error is uncorrelated with everything in } \phi_{t-1}.$$

Why is this useful?

AM show how to estimate the time-series process that the market believes  $X_t$  follows ('imbedded in stock prices'). Testing whether this process equals the true time-series process is equivalent to testing whether the market's forecast errors are predictable.

## Tests of capital market efficiency

Define  $R_t$  = asset return,  $y_t = R_t - E_{t-1}^M[R_t]$

Market efficiency implies that  $y_t$  is unpredictable:  $E[y_t | \phi_{t-1}] = 0$ .

Suppose we have any observable variable  $Z_{t-1}$  that is part of the information set  $\phi_{t-1}$ . Then the traditional approach to testing market efficiency is to test the restriction that  $\alpha = 0$  in:

$$y_t = Z_{t-1} \alpha + \varepsilon_t$$

This statement also means that  $y_t$  should only be correlated with information arriving at  $t$ . We can use this idea to derive a second test of market efficiency.

## Tests of capital market efficiency, cont.

Let  $X_t$  be any variable that is correlated, ex post, with returns. In the nonlinear system of equations:

$$X_t = Z_{t-1} \gamma + u_t$$

$$y_t = (X_t - Z_{t-1} \gamma^*) \beta + \varepsilon_t$$

where  $\text{cov}(u_t, Z_{t-1}) = 0$ , market efficiency implies that  $\gamma = \gamma^*$  ( $\beta$  is assumed not to be zero).

This is the nonlinear, cross-equation restriction analyzed by AM and applied by Dechow and Sloan.

*Note:* The coefficient estimates of the system will be consistent, not unbiased, and the test analyzed by AM is an asymptotic LR test.

## Questions

- (1) The cross-equation restriction is intuitive. But how do we know that NLLS actually produces the time-series process that investors believe  $X_t$  follows?
- (2) What is assumed about  $u_t$  in the equation  $X_t = Z_{t-1} \gamma + u_t$ ? In particular, does the restriction remain valid if the market uses more information than  $Z$  to predict  $X$ , so that  $E[u_t | \phi_{t-1}] \neq 0$  or possibly even  $E[u_t | Z_{t-1}] \neq 0$ ?
- (3) What is the relation between the two approaches to testing market efficiency?
- (4) How is the test implementable?

## Why does it work?

*An observation:* The LS estimate of  $y_t = (X_t - Z_{t-1} \gamma^*) \beta + \varepsilon_t$  identifies the  $\gamma^*$  that maximizes the squared correlation between  $y_t$  and  $X_t - Z_{t-1} \gamma^*$ .

Re-write  $X_t - Z_{t-1} \gamma^*$  as

$$X_t - Z_{t-1} \gamma + Z_{t-1} (\gamma - \gamma^*) = u_t + Z_{t-1} \delta.$$

where  $\delta = (\gamma - \gamma^*)$ . Holding  $\text{var}(y)$  constant, LS chooses  $\delta$  to maximize:

$$\frac{[\text{cov}(y_t, u_t + Z_{t-1} \delta)]^2}{\text{var}(u_t + Z_{t-1} \delta)} = \frac{[\text{cov}(y_t, u_t) + \delta \text{cov}(y_t, Z_{t-1})]^2}{\text{var}(u_t) + \delta^2 \text{var}(Z_{t-1})}$$

If the market is efficient, the numerator reduces to  $\text{cov}^2(y_t, u_t)$ , and it becomes clear that  $\delta = \gamma - \gamma^* = 0$  is the LS estimator.

*Important:* Assuming market efficiency, all that is necessary for  $\gamma = \gamma^*$  is that the covariance between  $u$  and  $Z$  equals zero, not  $E[u_t | \phi_{t-1}] = 0$ .

## Estimation

The 2nd equation can be estimated in a number of equivalent ways:

$$\begin{aligned} y_t &= (\mathbf{X}_t - \mathbf{Z}_{t-1} \gamma^*) \boldsymbol{\beta} + \varepsilon_t \\ &= \mathbf{X}_t \boldsymbol{\beta} - \mathbf{Z}_{t-1} \boldsymbol{\delta} + \varepsilon_t \end{aligned} \quad \text{with } \boldsymbol{\delta} = \gamma^* \boldsymbol{\beta} \quad (1)$$

$$\begin{aligned} &= [\mathbf{X}_t - \mathbf{Z}_{t-1} \gamma + \mathbf{Z}_{t-1} (\gamma - \gamma^*)] \boldsymbol{\beta} + \varepsilon_t \\ &= [(\mathbf{X}_t - \mathbf{Z}_{t-1} \gamma) + \mathbf{Z}_{t-1} \boldsymbol{\delta}] \boldsymbol{\beta} + \varepsilon_t \end{aligned} \quad \text{with } \boldsymbol{\delta} = \gamma - \gamma^* \quad (2)$$

$$= \mathbf{u}_t \boldsymbol{\beta} + \mathbf{Z}_{t-1} \boldsymbol{\delta} + \varepsilon_t \quad \text{with } \boldsymbol{\delta} = (\gamma - \gamma^*) \boldsymbol{\beta} \quad (3)$$

Three results are immediate:

- From (1), NLLS is identical to OLS for the unconstrained system
- From (2) and (3), we can test  $\gamma = \gamma^*$  by testing  $\boldsymbol{\delta} = 0$
- From (3), the cross-eq restriction is equivalent to  $\boldsymbol{\alpha} = 0$  in  $y_t = \mathbf{Z}_{t-1} \boldsymbol{\alpha} + \varepsilon_t$

## Dechow and Sloan (1997)

DS apply this methodology to testing whether the market has rational forecasts of growth in earnings and sales:

- LSV (1994) argue that the cross-sectional relation between average returns and ‘contrarian’ variables such as B/M and E/P arises because prices ‘reflect the naïve extrapolation of past growth in sales and earnings.’ For example, investors ignore mean reversion in earnings growth.
- Bauman and Dowen (1988) and La Porta (1995) argue that prices reflect the overoptimistic earnings forecasts of financial analysts.

These arguments imply that the earnings and sales expectations implicit in market prices are biased, and that we can find some instrument  $Z_{t-1}$  (past growth, analysts’ forecasts) that predicts returns.

## Dechow and Sloan (1997), cont.

DS estimate a pooled time-series / cross-sectional ‘stacked’ regression:

$$X_t = \gamma_0 + \gamma_1 Z_{t-1} + v_t,$$

$$r_t = \beta_0 + \beta_1 (X_t - \gamma_1^* Z_{t-1}) + \varepsilon_t$$

where  $X_t$  = growth in sales or growth in earnings

$Z_{t-1}$  = past growth rates or analysts’ forecasts

$r_t$  = buy and hold return

$\gamma_0$  is excluded from the return equation because it is not identified when an intercept is included. Therefore, DS test whether  $\gamma_1^* = \gamma_1$ . The test statistic is:

$$LR = 2n \log(SSR^c / SSR^u)$$

which is asymptotically distributed as  $\chi^2(1)$  under the null.

## Dechow and Sloan (1997), cont.

DS conclude that the evidence is consistent with investors relying on the biased estimates of analysts. The paper is innovative and useful, but ...

- I worry about the pooled TS/CS regression approach. The regression coefficients are unlikely to be constant across firms, which could have strange effects on the AM tests.

$$X_t = \gamma_0 + \gamma_1 Z_{t-1} + v_t,$$

$$r_t = \beta_0 + \beta_1 (X_t - \gamma_1^* Z_{t-1}) + \varepsilon_t$$

It is likely that the persistence of earnings differs across firms ( $\gamma_1$  varies), that the market's reaction to unexpected earnings depends on persistence ( $\text{cov}(\beta_1, \gamma_1) \neq 0$ ), and that there is a nonlinear relation between unexpected earnings and returns ( $\text{cov}(\beta_1, v_t) \neq 0$ ).

## Dechow and Sloan (1997), cont.

- DS assume that expected returns are constant (or at least unrelated to past earnings growth) and seem to be skeptical of the possibility that an equilibrium model can explain the results. However, the whole point of the rational stories is that ‘distressed’ firms have higher risks. In other words, past earnings proxy for risk and, consequently, expected return.
- A couple minor comments:

DS assume that  $E[v_t | \phi_{t-1}] = 0$ . This isn't necessary.

DS state the parameter estimates will be unbiased. No: consistent (at best).