Sample Answer

Fama and Schwert (1977) tested the usefulness of different assets as hedges against changes in expected and unexpected inflation. Their tests regressed asset returns on the t-bill rate (as a proxy for expected inflation given a hypothesis of a constant real rate) and the difference between the CPI and the t-bill rate. In the case of stock returns, they found that the coefficient on the expected inflation proxy was highly (and significantly) negative while the coefficient on unexpected inflation was negative but not reliably different than zero or one. They conclude that stocks do not hedge either expected or unexpected inflation well (although the second null hypothesis of hedging ability for unexpected inflation is not formally rejected by the data). Table 1 provides estimates of the same regression of stock returns on expected and unexpected inflation using different measures of expected and unexpected inflation for the Fama & Schwert time period (1953:01 to 1971:07).

Panel A provides a direct re-creation of the Fama & Schwert test, using the t-bill rate as expected inflation and the difference between the CPI and the t-bill rate as unexpected inflation. The estimates and standard errors are not identical (presumably due to slight data differences), but they are very close and the conclusions are identical. Wald statistics test the null hypotheses that β and γ (the coefficients on expected and unexpected inflation, respectively) each equals one as well as the null hypothesis that both equal one. Clearly, we can reject for β and for both together, but not for γ alone.

Estimates of expected and unexpected inflation for the results in Panel B are based on the following ARIMA (0,1,1) model which was fit to the in-sample data (standard errors in parentheses):

$$CPI_t - CPI_{t-1} = \underset{(.000015)}{.000018} - \underset{(.031)}{.892}\varepsilon_{t-1} + \varepsilon_t$$

The fitted values from this model are taken as the predicted changes in inflation and added to lagged inflation to produce the expected inflation series. CPI minus expected inflation equals unexpected inflation. The coefficient estimates in Table 1 for the stock regression based on these estimates differ somewhat from those in Panel A. The coefficient on expected inflation is still negative, but not as highly negative, while the coefficient on unexpected inflation is more negative than before. Both coefficients are independently reliably less than one based on the Wald test. These differences are not terribly surprising. Based on evidence we have seen in the past, the assumption of a constant real rate does not seem to describe the data very well. The ARIMA model is a much more realistic way to project expected and unexpected inflation. In Panel A, the variation in the interest rate most likely captures movements in both expected and unexpected inflation, while in Panel B these effects are more accurately separated. The coefficient on expected inflation in Panel A can be seen as a mixture of the true effects of expected and unexpected inflation, which is why it is more negative but with a higher standard error. The more accurate measurements in Panel B provide more powerful tests of the separate effects of expected and unexpected inflation.

The main problem with Panel B is that the expected and unexpected inflation measures use estimates of the ARIMA model from the entire sample period (thus expected inflation in the early parts of the period depend upon data which was not observable at the time). In order to correct for this problem, expected and unexpected inflation in Panel C were based on an ARIMA model whose coefficients were estimated out of sample. I used the sample period 1913:2 to 1952:12. The fitted ARIMA model for this period is:

$$CPI_t - CPI_{t-1} = .0000006 - .691\varepsilon_{t-1} + \varepsilon_t$$
$$(.000109) \quad (.033)$$

The parameter estimates are slightly different here. This model was then applied to the 1953 to 1971 data in order to generate a series of predicted changes in the CPI, which was added to lagged CPI to generate the expected inflation series. The unexpected inflation series again is actual CPI less expected inflation. In Panel C of Table 1, we see that the coefficient estimates on expected and unexpected inflation are pretty similar to those in Panel B, which is not surprising given that the ARIMA coefficients are not very different. However, rejection of the hypothesis that $\gamma = 1$ is less clear here. This probably results from the fact that the out of sample ARIMA model does not fit the in-sample data as well as the in-sample ARIMA model (by definition). This makes the inference somewhat less clear, although the problem of using information from the entire sample in forecasting inflation has been eliminated.

Panel D uses yet another approach to forecasting inflation. Inflation is forecast using an autoregressive process (lags 1, 2 and 12 are used since they were the only significant ones) which is estimated in a two-equation system along with the regression of stock returns on expected and unexpected inflation. The system is estimated using nonlinear least squares where the coefficients on lagged inflation in the expected and unexpected inflation expressions are constrained to equal those in the AR specification. In other words, the system is estimated as:

$$CPI = \delta + \rho_1 * CPI(-1) + \rho_2 * CPI(-2) + \rho_3 * CPI(-12)$$

$$Rt = \alpha + \beta * (\delta + \rho_1 * CPI(-1) + \rho_2 * CPI(-2) + \rho_3 * CPI(-12)) + \gamma * (CPI - (\delta + \rho_1 * CPI(-1) + \rho_2 * CPI(-2) + \rho_3 * CPI(-12)))$$

Where ρ_1, ρ_2, and ρ_3 must be equal across the two equations. The estimate of the CPI AR model turned out to be:

$$CPI = 0.001033 + 0.325229 * CPI(-1) + 0.185962 * CPI(-2) - 0.056717 * CPI(-12)$$
$$(0.002307) \quad (0.596452) \quad (0.375118) \quad (0.214811)$$

While the coefficients on these lags were significant in a single-equation autoregression (not shown), they are not significant here despite the fact that the point estimates are nearly identical. This presumably results from their lack of importance in the stock regression. The estimates of the stock return regression are given in Panel D of Table 1. The coefficients are very similar to those in Panel A, but the inference is opposite. The coefficient on unexpected inflation is reliably different than one, while the coefficient on expected inflation is not despite its large absolute value. This model provides a more realistic estimate of expected and unexpected inflation than the constant real rate assumption, but seems to be significantly less precise than the ARIMA model. However, in contrast to the ARIMA model approach, the systems approach allows for simultaneous estimation of the parameters of the model used to predict inflation, which may remove some endogeneity bias.

Panel E of Table 1 reproduces the results of Panel D but adds the t-bill rate, long-term corporate bond yields and the recession dummy variable as additional predictors of inflation. The addition of the other variables does not seem to significantly improve the model's fit. We are no longer able to strongly reject either independent null hypothesis of a coefficient equal to unity. By including the recession dummy, we again introduce an endogeneity bias since it is a post sample-period classification.

The cross-equation restrictions implied by the systems estimated for Panels D and E can be tested using a likelihood ratio test. These restrictions were outlined two paragraphs above: ρ_1, ρ_2, and ρ_3 must be equal across the two equations. However, given that both expected and unexpected inflation are included in the stock return regression, these restrictions are very weak. Since all of the independent variables appear twice in the stock return regression, the "restrictions" in reality do not constrain the coefficient estimates in the system very much versus "free" coefficient estimates in an unconstrained system. The LR statistics for these two models (computed as 2nlog(SSRr/SSRu), distributed Chi-squared with d.o.f. equal to the number of restrictions) are each less than .001, which could never lead to rejection of the null hypothesis of valid restrictions. A more relevant application of this test would be a regression of stock returns on expected or unexpected inflation alone (this would be more akin to a traditional Mishkin test situation), but this is not our object here.

Choosing between these different models depends upon your purpose. Using the constant real rate assumption is fairly unrealistic in terms of a description of the CPI data, but it is the only approach here that relies upon economic theory as opposed to statistical data fitting. The in-sample ARIMA approach probably fits the data the best, but it is not likely to appeal if we want to avoid endogeneity bias. The out-of-sample ARIMA model solves this problem, but does not fit the data as well. Finally, the systems methods account for any simultaneity in the determination of the coefficients in the model, but at a price in terms of the fit to the CPI data. In addition, the lack of significant real restrictions makes this model little different than an in-sample AR specification for CPI which would then be used to produce expected and unexpected inflation series for the stock return regression. Since the ARIMA model seems to fit the inflation data better, this approach does not seem very instructive. Finally, adding additional explanatory variables in the systems approach does not seem to improve things, but, of course, different choices of independent variables could lead to different results. This approach might appeal if we were on a "fishing expedition" to see what can be used to explain the empirical findings.

Table 2 provides results from the same estimation procedures applied to a more complete sample period, 1953:01 to 1995:10. This sample period was chosen to provide a more complete history in order to see whether the Fama & Schwert period was unique, while excluding the anomalous WWII and Great Depression data. The results indicate that the Fama & Schwert time period was somewhat unique, but not terribly so. Comparing Panels A, B and C of Table 2 to the same panels in Table 1, in each case the coefficient on unexpected inflation is now more negative than that on expected inflation. This could reflect less inflation predictability in the full period. The estimates for expected versus unexpected inflation are also closer in absolute value for each panel in Table 2 versus Table1. It seems that the Fama & Schwert period was unusual in the larger impact of the expected component of inflation. However, in each case the conclusion of the Wald tests indicates that all null hypotheses of stock returns being a perfect hedge against inflation can be rejected at the .01 level or better. This indicates that the negative relation between stock returns and both inflation components is more clear for the longer period.

Panels D and E of Table 2 tell a very different story. In each of these cases, the coefficient on expected inflation is positive, as compared to large negative coefficients in Table 1. However, similar to Table 1, these estimates are very imprecise, and they are not reliable different from either zero or one. The AR model does not seem to fit the data very well in the longer sample period, and perhaps even worse than in the early sample period (as evidenced by its strikingly different coefficient point estimates versus the other methods).

Table 1

Coefficients from Regressions of Stock Returns on Expected and Unexpected Inflation (standard errors in parentheses)
For Sample Period 1953:01 to 1971:07

Estimates of the Model:

$$R_{st} = \alpha + \beta \, \mathbf{E}[\rho_t \mid \phi_{t-1}] + \gamma \, \{\rho_t - \mathbf{E}[\rho_t \mid \phi_{t-1}]\} + \varepsilon_t$$

	α	β	γ	Adjusted R^2	Wald Stats (Probability in Parentheses) $\beta = 1$	$\gamma = 1$	$\beta = \gamma = 1$
A. Expected Inflation = INT, Unexpected Inflation = CPI - INT							
1953:01 - 1971:07	0.024 (0.006)	-5.591 (1.850)	-0.812 (1.235)	0.0326	12.689 (0.000)	2.153 (0.142)	14.600 (0.001)
B. Expected Inflation and Unexpected Inflation from **in sample** ARIMA (0,1,1) model for CPI inflation							
1953:01 - 1971:07	0.016 (0.004)	-3.254 (1.796)	-1.830 (1.227)	0.0139	5.614 (0.018)	5.319 (0.021)	10.154 (0.006)
C. Expected Inflation and Unexpected Inflation from **out-of-sample** ARIMA (0,1,1) model for CPI inflation							
1953:01 - 1971:07	0.017 (0.004)	-4.011 (1.551)	-1.335 (1.209)	0.0221	10.440 (0.001)	3.729 (0.053)	12.095 (0.002)
D. System Estimates, Expected and Unexpected Inflation from AR model (including lags 1, 2, 12) for CPI inflation							
1953:01 - 1971:07	0.020 (0.016)	-5.778 (7.789)	-1.536 (0.869)	0.0041	0.119 (0.730)	8.655 (0.003)	8.773 (0.012)
E. System Estimates, Expected and Unexpected Inflation from AR model (including lags 1, 2, 12) plus INT, LTCORP and BC for CPI inflation							
1953:01 - 1971:07	0.026 (0.029)	-8.706 (13.528)	-0.790 (0.913)	0.0193	0.515 (0.473)	3.845 (0.050)	4.360 (0.113)

Table 2

Coefficients from Regressions of Stock Returns on Expected and Unexpected Inflation (standard errors in parentheses)
For Sample Period 1953:01 to 1995:10

Estimates of the Model:

$$R_{st} = \alpha + \beta\, E[\rho_t \mid \phi_{t-1}] + \gamma\, \{\rho_t - E[\rho_t \mid \phi_{t-1}]\} + \varepsilon_t$$

	α	β	γ	Adjusted R^2	Wald Stats (Probability in Parentheses) $\beta = 1$	$\gamma = 1$	$\beta = \gamma = 1$
A. Expected Inflation = INT, Unexpected Inflation = CPI - INT							
1953:01 - 1995:10	0.016	-1.763	-2.123	0.0206	13.139	20.582	28.935
	(0.004)	(0.762)	(0.688)		(0.000)	(0.000)	(0.000)
B. Expected Inflation and Unexpected Inflation from **in sample** ARIMA (0,1,1) model for CPI inflation							
1953:01 - 1995:10	0.014	-1.184	-2.973	0.0266	9.747	25.777	32.254
	(0.003)	(0.699)	(0.783)		(0.002)	(0.000)	(0.000)
C. Expected Inflation and Unexpected Inflation from **out-of-sample** ARIMA (0,1,1) model for CPI inflation							
1953:01 - 1995:10	0.015	-1.361	-2.793	0.0245	11.871	23.409	31.112
	(0.003)	(0.685)	(0.784)		(0.001)	(0.000)	(0.000)
D. System Estimates, Expected and Unexpected Inflation from AR model (including lags 1, 2, 12) for CPI inflation							
1953:01 - 1995:10	0.009	0.223	-2.817	0.0244	0.109	49.358	49.467
	(0.009)	(2.355)	(0.543)		(0.741)	(0.000)	(0.000)
E. System Estimates, Expected and Unexpected Inflation from AR model (including lags 1, 2, 12) plus INT, LTCORP and BC for CPI inflation							
1953:01 - 1995:10	-0.001	3.159	-2.555	0.0259	0.095	39.586	39.680
	(0.025)	(7.017)	(0.565)		(0.758)	(0.000)	(0.000)