

Introduction

- Gordon Model (1962): $\frac{D}{P} = r - g$

r = constant discount rate, g = constant dividend growth rate.

- If rational expectations of future discount rates and dividend growth vary over time, so should the D/P ratio.
- Since we observe the dividends and returns ex-post, we can test the restrictions the dynamic Gordon model impose on the time-series of observed dividends and returns.

- Campbell and Shiller ask the question:

Do existing models of expected returns (discount rates) explain the variation in the dividend-to-price ratio after subtracting off dividend growth?

- Quick answer: NO

- Long answer:

1) The ex-post log dividend-to-price ratio is significantly different than the estimated log dividend-to-price ratio. The estimate of the log D/P is based on the rational expectations estimates of future discount rates and dividend growth rates based from the VAR.

2) The standard deviation of the present value of future expected discount rates is much smaller than the variation in log D/P not explained by expected log dividend growth.

Log D/P model

- $R_t = \frac{P_{t+1} + D_t}{P_t}$
- $\log(R_t) \equiv h_t \equiv \log(P_{t+1} + D_t) - \log(P_t)$
- Campbell and Shiller want a linear relationship, so they approximate h_t with a first-order Taylor series approximation (some error induced):
- $h_t \cong \mathbf{x}_t \equiv k + \mathbf{r} \log(P_{t+1}) + (1 - \mathbf{r}) \log(D_t) - \log(P_t) = k + \mathbf{r} p_{t+1} + (1 - \mathbf{r}) d_t - p_t$

- $\mathbf{d}_t \equiv d_{t-1} - p_t$ Don't forget these are all logs!
- Based on the Taylor approximation we get:
- $h_t \cong k + \mathbf{d}_t - r\mathbf{d}_{t+1} + \Delta d_t$
- Solving this equation, with the condition:
- $\lim_{i \rightarrow \infty} r^i \mathbf{d}_{t+i} = 0$, they get the relationship they want :
- $\mathbf{d}_t \cong \sum_{j=0}^{\infty} r^j (h_{t+j} - \Delta d_{t+j}) - \frac{k}{1-r}$.
- The log D/P is a function of the discounted value of future returns and dividend growth. All ex-post – NO ECONOMIC CONTENT!

- Add some theory about the behavior of ex-post returns and we can test the economic model of the D/P ratio.
- $E_t h_t = E_t r_r + c$,
- where E_t denotes a rational expectation formed by using information set I_t
- Now take the expectation of D/P ratio equation from the last slide from the based on the conditional information set I_t , which is available to agents at the beginning of t:
- $$d_t \cong E_t \sum_{j=0}^{\infty} r^j (h_{t+j} - \Delta d_{t+j}) + \frac{c - k}{1 - r}$$

Data and Unit Root Tests

- Two data sets are used: 1) S&P 500 index from 1871-1986, 2) Value-Weighted NYSE index from 1926-1986.
- T-bill rates and CPI inflation rates from Ibbotson Ass.
- Table 2.
- Need to check for unit roots. Why?
 - 1) Standard errors are no good if non-stationary regressors.
 - 2) Results are sensitive to stationarity assumptions when estimating.
- Table 3.

- Campbell and Shiller assume that the log D/P ratio, growth rates of real dividends and prices are stationary, so that log dividends and prices are cointegrated processes.
- Part of cointegration (1,1) means the variables are stationary in first differences. This turns out to be better than transforming the variables by removing a deterministic linear trend, because it induces biases.
- Campbell and Shiller (1987) go into the details of cointegration and what it means for the estimation of present value models. The VAR's I'm about to discuss are based on the 1987 paper. Campbell and Shiller (1988 JF) also use the VAR setup but include earnings.

VAR's

- Campbell and Shiller want to forecast future discount rates and dividend growth rates and compute an implied D/P ratio, then compare the implied movements in the D/P ratio to the observed movements in the D/P ratio.
- They use a VAR to estimate the expected future discount rates and expected future dividend growth.
- They include the log D/P ratio as a variable in the VAR to generate forecasts. The log D/P summarizes all relevant info in the market so they aren't missing anything, and the forecasts exactly equal the D/P ratio.

- Campbell and Shiller assume market observes y_t , a vector of state variables and y_t follows a linear stochastic process.
- They choose a x_t , so that it is the smallest that allows them to test the restrictions of the D/P model.
- $x_t = [\mathbf{d}_t, r_{t-1} - \Delta d_{t-1}]'$
- They also assume x_t can be written as a VAR with p lags. They make a transformation so they can rewrite the VAR in first order autoregression. The resulting VAR system is:
- $z_t = Az_{t-1} + \mathbf{u}_t$

- Conditioning on the econometrician's information set (H_t) they get:

- $$\mathbf{d}_t \cong E \left[\sum_{j=0}^{\infty} \mathbf{r}^j (h_{t+j} - \Delta d_{t+j}) \mid H_t \right] \equiv \mathbf{d}'_t$$

- Rewriting this equation in terms of the VAR forecasts:

- $$\mathbf{d}_t = e_1' z_t = \sum_{j=0}^{\infty} \mathbf{r}^j e_2' A^{j+1} z_t \equiv \mathbf{d}'_t$$

- which imposes the following restriction which can be tested with a Wald test,

- $e1' = \sum_{j=0}^{\infty} r^j e2' A^{j+1} = e2' A(I - rA)^{-1}$
- A test which is algebraically equivalent is:
- $e1'(I - rA) - e2' A = 0$
- The previous x_t does not separate discount rates and dividend growth. If x_t has three elements we can test the separate effects of each variable. The new restriction is:
- $e1' z_t = \sum_{j=0}^{\infty} r^j (e3' - e2') A^{j+1} z_t \equiv d_t'$, and
- $e1'(I - rA) - (e3' - e2') A = 0$

- Key relationship is:

- $\mathbf{d}_t = \mathbf{d}'_t \equiv \mathbf{d}'_{rt} + \mathbf{d}'_{dt}$

- Finally, if we observe consumption growth or variance in stead of the discount rate itself, consumption growth or variance will be in the x_t vector and the following restrictions must hold.

- $\mathbf{d}_t = \mathbf{d}'_t \equiv \mathbf{d}'_{rt} + \mathbf{d}'_{dt} \equiv \mathbf{a}e3' A(I - \mathbf{r}A)^{-1} z_t - e2' A(I - \mathbf{r}A)^{-1} z_t$, and

- $e1'(I - \mathbf{r}A) - (\mathbf{a}e3' - e2')A = 0$

- where $r_r = \mathbf{a}\Delta c_t$ or $r_r = \mathbf{a}V_t$, and \mathbf{a} is the relative risk aversion coefficient and is estimated from the restrictions.

Empirical Results

- Tables 4-7

Conclusion, Problems, and Extensions

Main Results:

- Log D/P ratio does move rationally with expected future growth in dividends.
- Measures of discount rates in the paper do not explain stock-price movements very much. (Granger causality tests are not sig.)
- Variation in dividend growth and discount rates do not fully explain movements in dividends. - “Dividends are too variable”

Possible Problems:

- Taylor series approximation induces bias. Campbell and Shiller (1988) show that the bias is not big enough to explain the results.

- Use a different measure of discount rates.
- It is possible that the processes are not stable, linear stochastic processes.

Recent Extensions:

- Bekaert and Grenadier (2000), Vuolteenaho (2000), Young (2000)

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