SWITCHING REGRESSION MODELS AND ESTIMATION

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Outline

Switching Regression Models

- Model setting
- Motivation
- Estimation (Two-stage method)
- Variations
  - Censored models
  - Models with self-selectivity

Switching Regression Models — Setting

$$\text{Regime 1:} \quad y_i = b_1' X_{1i} + u_{1i} \quad \text{iff} \quad g' Z_i \geq u_i \quad (1)$$

$$\text{Regime 2:} \quad y_i = b_2' X_{2i} + u_{2i} \quad \text{iff} \quad g' Z_i < u_i \quad (2)$$

We assume that u_{1i}, u_{2i}, and u_i have a trivariate normal distribution, with mean vector zero and covariance matrix

$$\Sigma = \begin{bmatrix} s_1^2 & s_{12} & s_{1u} \\ & s_2^2 & s_{2u} \\ & & 1 \end{bmatrix} \quad (3)$$

Switching Regression Models — Applications

- The Union-nonunion-wage model (Lee, 1978)
- The Housing-demand model (Trost, 1977)
- Disequilibrium Market model (Fair and Jaffee, 1972)
- The Labor-supply model (Heckman, 1974)
- The Labor-supply model (Gronau, 1974)
- Needs vs. Reluctance model (Polakoff and Sibler, 1967)

Switching Regression Models — Estimation (1)

OLS Estimation

$$E(u_{1i} \mid u_i \geq g'Z_i) = E(s_{1u} u_i \mid u_i \leq g'Z_i) = -s_{1u} \frac{f(g'Z_i)}{f(g'Z_i)} \qquad (4)$$

$$E(u_{2i} \mid u_i \geq g'Z_i) = E(s_{2u} u_i \geq g'Z_i) = s_{2u} \frac{f(g'Z_i)}{1 - f(g'Z_i)} \qquad (5)$$

OLS estimation is inappropriate.

Switching Regression Models — Estimation (2)

Maximum Likelihood Estimation

$$L(b_1, b_2, s_1^2, s_2^2, s_{1u}, s_{2u})$$

$$= \prod \left[\int_{-\infty}^{g'Z_i} g(y_i - b_1' X_{1i}, u_i) du_i \right]^{I_i} \left[\int_{g'Z_i}^{\infty} f(y_i - b_2' X_{2i}, u_i) du_i \right]^{1-I_i} \quad (6)$$

- The ML estimates can be shown to be consistent and asymptotically efficient
- The estimation can be cumbersome

Switching Regression Models — Estimation (3)

Tobit Models

$$y_i = b' X_i + u_i \qquad \text{if RHS>0}$$

$$y_i = 0 \qquad \text{otherwise} \qquad (7)$$

Switching Regression Models — Estimation (4)

Two-stage method for Tobit Models

$$E(u_i \mid u_i > -b' X_i) = s \frac{f_i}{\Phi_i} \qquad (8)$$

$$\Rightarrow \quad y_i = b' X_i + s \frac{f_i}{\Phi_i} + v_i \qquad (9)$$

where $E(v_i) = 0$

Switching Regression Models — Estimation (5)

Two-stage method for Tobit Models

Stage1: Get the ML estimates of b / s using probit model, and then get estimated values of unknown variables in the expected value of residuals

$$I_i = \begin{cases} 1 & if\ y_i \geq 0 \\ 0 & otherwise \end{cases} \qquad (10)$$

Likelihood function

$$L = \prod_{I_i = 1} [\Phi_I]^{I_i} [1 - \Phi_i]^{1 - I_i} \qquad (11)$$

$$\Rightarrow \widehat{b / s} \Rightarrow \hat{f}_i, \hat{\Phi}_i$$

Switching Regression Models — Estimation (6)

Two-stage method for Tobit Models

Stage 2: Get consistent estimates of b and s by estimating the original equation by OLS, using $\hat{f}_i / \hat{\Phi}_i$ in place of f_i / Φ_i as an explanatory variable

Switching Regression Models — Estimation (7)

Two-stage method for Tobit Models

Or :

$$E(y_i) = \Pr ob(y_i > 0) \cdot E(y_i \mid y_i > 0) + \Pr ob(y_i \le 0) \cdot E(y_i \mid y_i \le 0)$$

$$= \Phi_i (b' X_i + s \frac{f_i}{\Phi_i}) + 0$$

$$= b'(\Phi_i X_i) + s f_i \qquad (12)$$

Switching Regression Models — Estimation (8)

Two-stage Estimation Method (Heckman, 1974; Lee, 1976)

Essential Features

- First obtain the expected values of the residuals that are truncated. Estimate the unknown parameters in the expected values by a probit model.
- Introduce the estimated values of these variables into the original equation and estimate it by proper least squares

→ The two-stage estimates are consistent

→ The estimations is simpler compared with ML

→ The two-stage estimates can be used as initial values for iteration of ML estimation.

Switching Regression Models — Variation (2)

Censored Models

- Labor-supply model (Gronau, 1974)
- Needs vs Reluctance model (Polakoff and Sibler, 1967)

$$g\,'Z_i = \frac{b_1' X_{1i} - b_2' X_{2i}}{s} \quad and \quad u = \frac{u_2 - u_1}{s} \qquad (16)$$

where,

$$s^2 = Var(u_2 - u_1) = s_1^2 + s_2^2 - 2s_{12} \qquad (17)$$

Switching Regression Models — Variation (1)

Censored Models

$$y_1 = b_1^{'} X_1 + u_1 \qquad (13)$$

$$y_2 = b_2^{'} X_2 + u_2 \qquad (14)$$

and we observe

$$\begin{aligned} y &= y_1 && \text{if } y_1 \geq y_2 \\ y &= 0 && \text{otherwise} \end{aligned} \qquad (15)$$

Switching Regression Models — Variation (3)

Identification Conditions (Nelson, 1975)

1. $s_{12} = 0$
2. There is at least one variable in X_1 not included in X_2. (In the context of the labor-supply model, there is at least one explanatory variable in the market-wage function not included in the reservation-wage function.)

Switching Regression Models — Variation (4)

Self-selection Models

- Occupation decision model (Roy, 1951)
- Labor Supply by Women (Gronau and Lewis, 1974)
- Housing demand model (Lee and Trost, 1978)
- Evaluation of the benefits of social programs

Switching Regression Models — Variation (5)

Self-selection Models

$$y_{1i} = X_i b_i + u_{1i} \quad \text{(for participants)} \quad (18)$$

$$y_{2i} = X_i b_2 + u_{2i} \quad \text{(for nonparticipants)} \quad (19)$$

$$I_i^* = Z_i g + e_i \quad \text{(participation decision function)} \quad (20)$$

$$I_i = 1 \quad iff \; I_i^* > 0 \quad (21)$$

$$I_i = 0 \quad iff \; I_i^* \leq 0 \quad (22)$$

The observed y_i is defined as

$$y_i = y_{1i} \quad iff \; I_i = 1 \quad (23)$$

$$y_i = y_{2i} \quad iff \; I_i = 0 \quad (24)$$

$$Cov(u_{1i}, u_{2i}, e_i) = \begin{bmatrix} s_{11} & s_{12} & s_{1e} \\ s_{12} & s_{22} & s_{2e} \\ s_{1e} & s_{2e} & 1 \end{bmatrix} \quad (25)$$

Switching Regression Models — Variation (6)

Self-selection Models

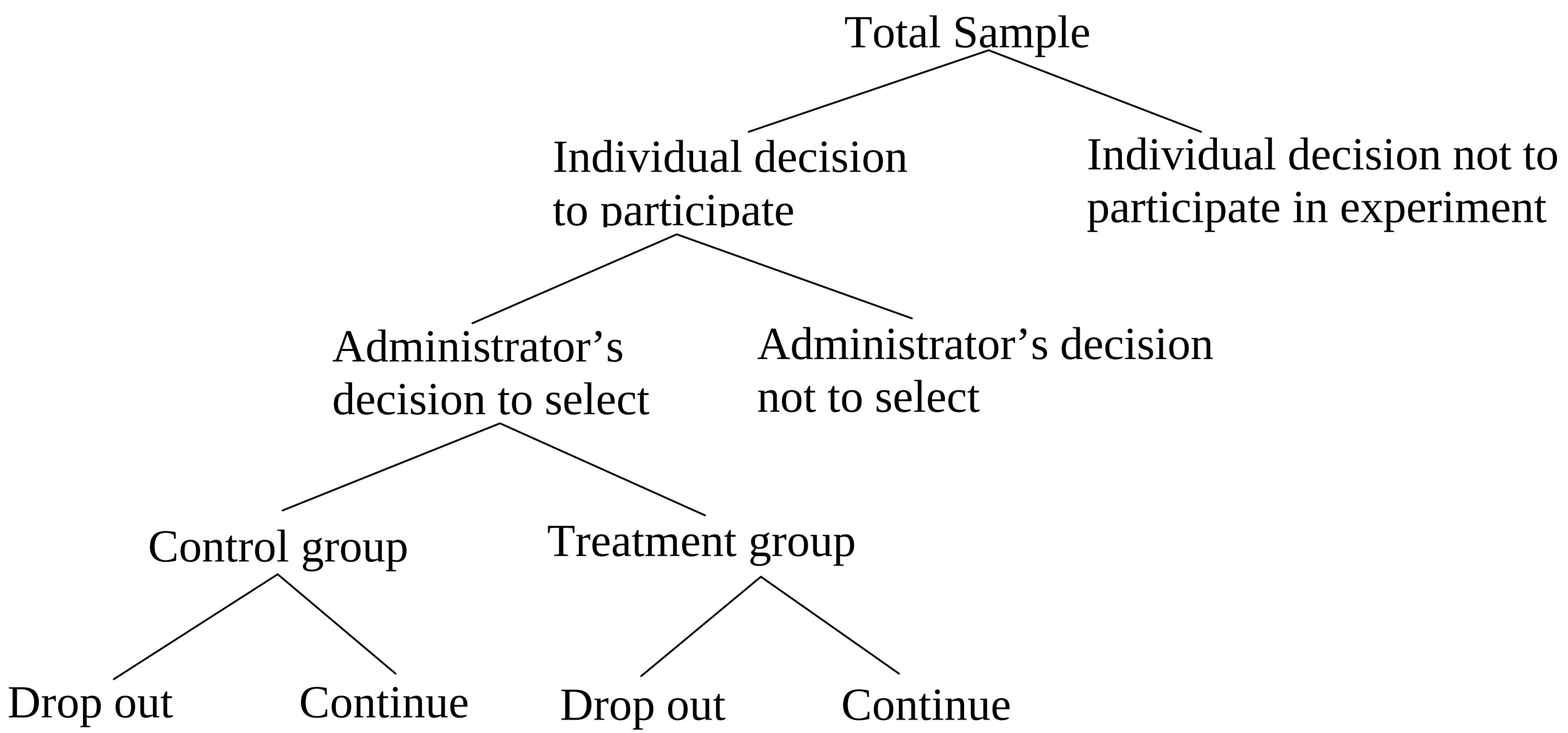
How to measure the benefit of the program?

$$y_{1i} - E(y_{2i} \mid I_i = 1) = y_{1i} - X_i b_2 + s_{2e} \frac{f(Z_i g)}{\Phi(Z_i g)} \qquad (26)$$

$$E(y_{1i} \mid I_i = 1) - E(y_{2i} \mid I_i = 1)$$
$$= X_i (b_1 - b_2) + (s_{2e} - s_{2e}) \frac{f(Z_i g)}{\Phi(Z_i g)} \qquad (27)$$

Switching Regression Models — Variation (7)

Self-selection Models



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