Drawing Inferences from Statistics Based on Multiyear Asset Returns

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Motivation

Fama and French (1988), Poterba and Summer (1988) document significant negative correlations in multi-year stock returns, using conventional asymptotic theory to asses the degree of statistic significance

Motivation

How powerful is the classical asymptotic theory for data that has a lot of overlapping observations, so that there is not a lot of independent information in multi-year statistics?

Novel Approach

Conventionally, the overlap in multiyear data is treated as fixed, so that as the time span increases, the ratio of the overlap to the sample size approaches zero.

Novel Approach

Richardson and Stock propose that the overlap grows with the time span, so that as sample size increases, the ratio of the overlap to the sample size approaches fixed constant

$$J/T \rightarrow d$$

Variance Ratio Statistics

Poterba and Summers (1988) use Variance Ratios to search for mean reversion in stock returns. For a two-period return:

$$VR(2) = \frac{\operatorname{var}(\tilde{r}_t + \tilde{r}_{t-1})}{2\operatorname{var}(\tilde{r}_t)}$$

$$= \frac{2\operatorname{var}(\tilde{r}_t) + 2\operatorname{cov}(\tilde{r}, \tilde{r}_{t-1})}{2\operatorname{var}(\tilde{r}_t)}$$

Variance Ratio Statistics

- Return generating process

$$r_t = m + e_t$$

- where

$$E(e_t / e_{t-1}, e_{t-2}, \ldots\ldots, e_1) = 0$$

$$h_t = E(e_t^2 / e_{t-1}, e_{t-2}, \ldots\ldots, e_1)$$

$$E\{(1/T)\sum_{t=1}^{T} h_t \rightarrow s^2$$

Variance Ratio Statistics

2 Variance Ratios are considered:

(1) $\Im_r(J)$ with non-overlapping data

(2) $\mathrm{M}_r(J)$ with overlapping data

Variance Ratio Statistics

$$\Im_r(J) = \frac{\left\{ \left(\frac{1}{T/J} \right) \left(\frac{1}{J} \right) \left[\begin{array}{l} \left(x_J(J) - J\hat{m} \right)^2 + \left(x_{2J}(J) - J\hat{m} \right)^2 \\ + \ldots\ldots + \left(x_T(J) - J\hat{m} \right)^2 \end{array} \right] \right\}}{\hat{s}_r^{\,2}}$$

- where $x_t(J) = \sum_{i=0}^{J-1} r_{t-i}$ $\qquad \hat{m} = \frac{1}{T} \sum_{t=1}^{T} r_t$

$$\hat{s}_R^{\,2} = \frac{1}{T} \sum_{t=1}^{T} (r_t - \hat{m})^2$$

Variance Ratio Statistics

$$\mathrm{M}_r(J) = \frac{\left\{ \dfrac{1}{JT} \sum_{t=J}^{T} (x_t(J) - J\hat{m})^2 \right\}}{\hat{s}_r^{\,2}}$$

Variance Ratio Statistics

Under the asymptotic fixed J assumption, assuming i.i.d normal stock returns:

$$\sqrt{T}(\Im_r(J)-1)\xrightarrow{d}N(0,2(J-1))$$

$$\sqrt{T}(\mathrm{M}_r(J)-1)\xrightarrow{d}N(0,\frac{2(2J-1)(J-1)}{3J})$$

Variance Ratio Statistics

Under the $J/T \to d$ assumption:

$$\Im_r(J) \xrightarrow{d} dc^2_{1/d}$$

- is not consistent
- has a nondegenerate limiting distribution that is non-normal
- variance of $x_t(J)$ tends to infinity

Variance Ratio Statistics

Under the $J/T \rightarrow d$ assumption:

$$M_r(J) \xrightarrow{d} \frac{1}{d}\int_d^1 Y_d(l)^2 \, dl$$

– where $Y_d(l) = W(l) - W(l-d) - dW(1)$ and W(.) is the standard Brownian motion

$M_r(J)$ has a non-normal limiting distribution variance of

Multiyear Autocorrelation Statistics

Fama and French (1989) examine the autocorrelation of multiyear returns using:

$$x_{t+J}(J) = a(J) + b(J)x_t(J) + e_{t+J}(J)$$

Multiyear Autocorrelation Statistics

- Under the J fixed assumption, the distribution of $\hat{b}(J)$ is approximated by a normal with a variance that increases with J.

- Under the $J / T \to d$ assumption, $\hat{b}(J)$ has the same distribution as a functional of a Brownian motion

Multiyear Autocorrelation Statistics

- To test the hypothesis that several multiyear correlations simultaneously equal zero, an F-test is used.
- Under the $J/T \to d$ assumption, F does not have the usual chi-square distribution
- The distribution of F can be evaluated numerically

Multiyear Autocorrelation Statistics

- To test whether whether the sum of the individual multiyear correlations are zero, the following statistic is computed:

$$r_t = \sum_{i=1}^{K} \hat{b}(J_i)$$

where

$$r_T \Rightarrow \sum_{i=1}^{K} b_{d_i}^{*}$$

with $b_{d_i}^{*}$ being the limiting statistic of $\hat{b}(J)$

Monte Carlo Evidence

- The asymptotic critical values of the statistics proposed can be obtained using Monte Carlo simulations
- The Monte Carlo results converge quickly with T; the $J / T \to d$ asymptotic distribution is good even for small T
- The results from the fixed J approximations differ substantially from the Monte Carlo results

Monte Carlo Evidence

- For the F-test of zero multi-year correlations, the Monte Carlo critical values differ substantially from the conventional chi-square distribution, the latter rejecting the null to often

- For the test of the sum of the multiyear correlations, the $J/T \to d$ approximation is better than the fixed J approximation

Fama and French (1988) re-examined

- The estimates of the autocorrelations of multiyear returns from Fama and French are consistent with mean reversion in stock returns
- This is not confirmed here; only 3 out of 96 point estimates reject the null of no autocorrelation (Fama and French find 19 significant point estimates)
- The hypothesis that the sum of the point estimates is zero cannot be rejected either

Poterba and Summers (1988) re-examined

- The evidence using the $J/T \to d$ approximations is mixed.
- The average of the point estimates of the variance ratios are consistent with mean reversion (10% level)
- The F-statistic reject the null of no autocorrelation only for 2 cases out of 8

Conclusion

- The $J/T \to d$ approximation provide a substantial improvement of the standard fixed J approximations
- The evidence in favor of mean reversion in stock returns is a lot less strong when the $J/T \to d$ approximations are used to compute limiting distributions

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