This will introduce you to the idea of cross-sectional tests of the CAPM. You should download the monthly returns of 25 equal-weighted portfolios formed on size and book to market, the size (ME) of the portfolios, and a proxy for the market portfolio ($R_M$) from 1964 until 2011 from the website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/. You may use any statistical package to do your work. Your report should include tables of results, not copies of computer output. All tables and charts should have legends and explanations. Answers (excluding tables and figures) should be typed and a maximum of three pages long (Times New Roman 11p font or larger, 1.5 line spacing, 1 inch margins).

a) (10 points) Consider the cross-sectional regression,

\[ R_i = \gamma_0 + \gamma_M \beta_{iM} + \eta_i, \]

where $\gamma_0$ and $\gamma_M$ are regression parameters and $\beta_{iM} = \text{cov}(R_i, R_M)/\sigma^2(R_M)$. If the CAPM holds, then what should $\gamma_0$ and $\gamma_M$ equal (for both the Sharpe/Lintner and Black versions)?

b) (20 points) Estimate $\gamma_0$ and $\gamma_M$ using the approach pioneered by Fama and MacBeth. The following is a brief outline of the procedure:

- Estimate $\beta_{iM}$ for each portfolio (denote the estimate $b_{iM}$). Assume that the betas do not change over time; hence, you can estimate the betas using full-period OLS regressions.

- Each month estimate the cross-sectional regression, $R_{it} = \gamma_{0t} + \gamma_{Mt} b_{iM} + n_{it}$. Note: the estimated beta is the same for every time period.

- Compute the time series average of the estimates of $\gamma_0$ and $\gamma_M$. In addition, compute the standard error and t-stat of the time series averages.
Can you reject the hypothesis that the proxy for the market portfolio is mean variance efficient? Why or why not? Comment on the estimated magnitude of the relation between the expected return of a portfolio and its beta.

c) (20 points)
Estimate the cross-sectional regression, \( \text{avg}(R_i) = \gamma_0 + \gamma_M b_{iM} + n_i \). Are the estimates of \( \gamma_0 \) and \( \gamma_M \) different than the average estimates in part b? Are the standard errors of the estimates of \( \gamma_0 \) and \( \gamma_M \) different than the standard errors of the average estimates in part b? Why or why not? Which method is superior? Why?

d) (10 points)
In part b) we assumed that the portfolio betas do not change over time. Discuss the validity of this assumption in this specific context.

e) (10 points)
Consider the cross-sectional regression,

\[
R_i = \gamma_0 + \gamma_M b_{iM} + \gamma_{ME} \ln(ME) + \eta_i
\]

where \( \gamma_0, \gamma_M, \) and \( \gamma_{ME} \) are regression parameters, \( b_{iM} = \frac{\text{cov}(R_i, R_M)}{\sigma^2(R_M)} \), and ME is the (appropriately lagged) size. If the CAPM holds, what should \( \gamma_{ME} \) equal? If the CAPM holds, what should \( \gamma_{ME} \) equal in the cross-sectional regression

\[
R_i = \gamma_0 + \gamma_{ME} \ln(ME) + \eta_i
\]

f) (30 points)
Estimate \( \gamma_0, \gamma_M, \) and \( \gamma_{ME} \) in regression (2) using the Fama MacBeth procedure. The following is a brief outline of the procedure:

- Estimate \( b_{iM} \) for each portfolio. Assume that the betas do not change over time; hence, you can estimate the betas using full-period OLS regressions.

- Each month estimate the regression, \( R_{it} = \gamma_{0t} + \gamma_{Mt} b_{iM} + \gamma_{MEt} \ln([ME]_{t-1}) + \eta_{it} \).

- Compute the time series average of the estimates of \( \gamma_{0t}, \gamma_{Mt}, \) and \( \gamma_{MEt} \). In addition compute the standard error and t-stat of the time series averages.

Can you reject the hypothesis that the proxy for the market portfolio is mean variance efficient? Why or why not? If you can reject the hypothesis, does that necessarily constitute a rejection of the CAPM? Why?