Preemptive Bidding and the Role of the Medium of Exchange in Acquisitions

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ABSTRACT

The medium of exchange in acquisitions is studied in a model where (i) bidders' offers bring forth potential competition and (ii) targets and bidders are asymmetrically informed. In equilibrium, both securities and cash offers are observed. Securities have the advantage of inducing target management to make an efficient accept/reject decision. Cash has the advantage of serving, in equilibrium, to "preempt" competition by signaling a high valuation for the target. Implications concerning the medium of exchange of an offer, the probability of acceptance, the probability of competing bids, expected profits, and the costs of bidders are derived.

In structuring its offer to acquire a firm, an acquirer must, among other things, determine the medium of exchange of the offer. That is, an acquirer must choose whether the payment will be in the form of cash, debt, equity, or some combination. With symmetric information, no transactions costs, and no taxes, the medium of exchange is irrelevant. This is not the case, though, if these assumptions are not satisfied. This paper studies the role of the medium of exchange in acquisitions in a setting in which there is asymmetric information between a target and competing bidders.

The focus of the paper is on the role of the medium of exchange in preempting competition. Consider a bidder that studies the profitability of an acquisition. If it makes a bid, other potential bidders will observe the bid, learn of the potentially profitable acquisition, and perhaps compete for it. A preemptive bid may be a way to eliminate this competition. Suppose a competing bidder's expected payoff is decreasing in the initial bidder's valuation for the target. When bidding against an initial bidder with a high valuation, a competitor may face a low probability of winning the bidding and a low expected payoff given that it does win. In this case, if the initial bidder could signal a sufficiently high valuation, it could deter the competition. As Fishman [7] and P'ng [18] have shown, a high bid can signal a high valuation and thus serve to preempt competition. Both studies, however, deal only with cash offers. (See also Giammarino and Heinkel [9] and Khanna [14].)

A key difference between a cash offer and a (risky) securities offer is that a security's value depends on the profitability of the acquisition, while the value of cash does not. In the studies cited above, bidders, but not the target, have private

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information on the profitability of the acquisition, and thus on the value of securities offers. Since no party has private information on the value of cash, all bidders offer cash. This is because an equity or risky debt offer would be presumed to have a low value, for, if the bidder knew the securities had a high value, it would have offered cash. The analysis here allows the target, in addition to bidders, to observe private information on the profitability of an acquisition. This is an important possibility to consider. Target management would likely have the best information on the target's physical assets and contractual commitments.

Securities offers then become a relevant alternative to cash offers. Consider offering a high payment if the target's information indicates a profitable acquisition, and a low payment otherwise. This would induce the target to make an efficient, given its information, accept/reject decision. If, however, the target's information is not verifiable, this offer is not feasible. An alternative, though, is a securities offer. Rather than making the offer contingent on the target's information about future cash flows, a securities offer's value is contingent on the cash flows themselves. If structured properly, a securities offer also induces the target to make an efficient accept/reject decision. In contrast, the value of a cash offer is not contingent on the future cash flows of the acquired target. Thus, the target can make its accept/reject decision independently of any information on these cash flows, and a cash offer cannot induce an efficient accept/reject decision.

A model of preemptive bidding is developed. In equilibrium, securities are offered by lower valuing bidders and cash by higher valuing bidders. The advantage of a securities offer is its ability to induce an efficient accept/reject decision on the offer. The advantage of a cash offer is that, in equilibrium, it serves to preempt potential competition by signaling a high valuation. Among the implications of the analysis are:

1. An initial bidder's expected payoff is lower if the medium of exchange of its initial offer is securities as compared to cash.
2. The probability that competing bids will be observed is higher after an initial securities offer as compared to an initial cash offer.
3. The probability that target management will reject an offer is higher if the medium of exchange is securities as compared to cash.
4. The higher the cost of studying a target, the more likely that an initial bidder's offer is cash and the less likely that there is a multiple bidder contest.

Hansen [12] (in independent work) also studies the choice of medium of exchange in acquisitions. As is the case here, the benefit of a securities offer is that it offers a state-contingent payment to the target. The benefit of a cash offer, however, differs between models. In Hansen, bidders have private information on their own premerger values, and, in equilibrium, bidders offer cash if their equity is relatively undervalued. Here, cash offers are made to signal a high valuation for the target, in order to preempt a potential competing bidder. A difference in predictions between the two models concerns a target's share price response to the outcome of an offer. The model here predicts a target share price
increase (decrease) if an offer is accepted (rejected), while, in Hansen, the reverse is predicted. Empirically, Dodd [5] reports results that are consistent with the predictions here. In addition, the analysis here addresses issues concerning the degree of competition in acquisitions markets. Whether an initial bidder faces competition for the target is endogenously determined. (In Hansen, there is assumed to be one bidder.) This leads to predictions on the interrelationships between the medium of exchange of an offer, the number of bidders, the profits from an acquisition, and the costs of studying a target.

Section I sets out the model, and Section II analyzes the equilibrium. Section III discusses the possibility of advance disclosure of the target’s private information. If bidders are not deterred from studying the target altogether, a target’s expected payoff is higher with advance disclosure. Section IV discusses some additional implications of the model.

I. The Model

For a given target there are two potential bidders. Assume that the target and both bidders are all-equity firms with risk-neutral shareholders. Also, target and bidder managements are assumed to maximize the wealth of their shareholders, and there is no collusion.\(^1\) It is assumed that the target accepts the offer with the highest value if it is at or above a known reservation price. Though the analysis goes through with any reservation price, it is assumed equal to the target’s prebid market value, denoted \(v_0\).\(^2\)

At a known cost \(h_i > 0\), bidder \(i\) can observe a private signal, \(s_i\), which conveys information on its own valuation for the target but is independent of the other bidder’s valuation. In addition, the target costlessly observes a private signal, \(s_0\), which conveys information on both bidders’ valuations. Assume that \(\tilde{s}_0, \tilde{s}_1,\) and \(\tilde{s}_2\) are mutually independent. Also, assume that there is no private information on the target value if unacquired. Let \(f_i(\cdot)\) denote the density function of \(\tilde{s}_i\) (\(i = 1, 2\)), where \(f_i(s_i) > 0\) if and only if \(s_i \in [l_i, h_i]\). Assume that \(\tilde{s}_0\) has a two-point distribution: \(\tilde{s}_0 = \alpha(\beta)\) with probability \(1 - \gamma(\gamma)\), where \(0 < \gamma \leq 1\). Denote bidder \(i\)’s valuation for the target as \(v_i = v^\alpha(s_i)\), for \(s_0 = \alpha\), and \(v_i = v^\beta(s_i)\), for \(s_0 = \beta\). It is assumed that \(v^\alpha(s_i)\) and \(v^\beta(s_i)\) are increasing in \(s_i\), and \(v^\beta(s_i) > v^\alpha(s_i) \geq 0\), for

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1 If target management has private information on the value of a securities offer (the case studied here), a conflict of interest between management and shareholders may provide an additional (to this analysis) benefit to cash offers. Say target management has some unobserved preference for control. Then it may reject a securities offer, claiming it has a low value when it actually has a high value. Cash has the advantage that shareholders need not rely on management to value it for them. For related work, see Baron [2].

2 Suppose the problem is repeated, period after period, until the target is acquired. With an infinite horizon and stationary environment, the market value of the target, \(w_t\), is the same at the start of every period. Further, this is the target’s reservation price in a take-it-or-leave-it offer. Note that, in an auction, the target receives the second highest valuation and the high bidder receives the difference in bidders’ valuations. If it were the case that the target and high bidder would bargain over this difference in valuations, then the signaling stage of the problem would become more complicated. An initial bidder would be simultaneously signaling to both the potentially competing bidder and the target. The target’s beliefs become important because they affect the outcome of the later-stage bargaining game.
all $s_i$. Moreover, assume that $u^*(h) < v_0$. Thus, since $v_0$ is the minimum acquisition price, neither bidder can profitably acquire the target if $s_0 = \alpha$. Also assume that $E[u^*(s_i)] < v_0$. This implies that it is not profitable for bidder $i$ to bid without observing $s_i$.

The problem unfolds as follows. Bidder 1 exogenously learns of a potentially profitable acquisition. Assume that only a few firms are potentially profitable acquisitions, so that studying random firms is not profitable. Once bidder 1 learns of a target, it can pay $k_1$ to observe $s_1$, and then perhaps make an offer. If bidder 1 does not make an offer, the problem ends with the target remaining known only to bidder 1. If bidder 1 does make an offer, the target is identified. Bidder 2, after observing this offer, determines whether to compete for the target. After bidder 2 makes its decision, the target costlessly observes $s_0$. (The possibility of observing and disclosing $s_0$ prior to any bidding is discussed in Section III.)

If bidder 2 competes, it pays $k_2$ to observe $s_0$. Then a competitive open auction follows: i.e., the high offer rises until one bidder remains. If bidder 2 does not compete, bidder 1's initial offer is the high offer. In either case, the target accepts the high offer if its value equals or exceeds $v_{10}$, and rejects it otherwise. If an offer is accepted, control of the target changes, and the new target value is realized.

The mediums of exchange considered are cash and debt, and offers are for all target shares. If a cash offer $p$ from bidder $i$ is accepted, bidder $i$ receives $v_i - p$, and the target receives $p$. The debt considered is discount debt backed by the target's assets. If a debt offer from bidder $i$ with face value $p$ is accepted, bidder $i$ receives $\max(v_i - p, 0)$, and the target receives $\min(v_i, p)$. If the target under bidder $i$'s control is worth $p$ or more, the debt liability is satisfied, and, if it is worth less than $p$, the bidder defaults on the debt and turns over the assets. In what follows, offers of cash or debt with face value $p < v_0$ can be ignored, since they would always be rejected. Also, assume that bidder 1 never offers debt with face value $p > v^*(s_1).$

A debt offer with face value $p$ is worth $\min[u^*(s_0), p] > v_0$ if $s_0 = \beta$, and $\min[u^*(s_0), p] < v_0$ if $s_0 = \alpha$. Thus, the target will accept (reject) the offer if $s_0 = \beta$ ($s_0 = \alpha$). This is an efficient decision rule in that the target accepts the offer only if its private information indicates a profitable acquisition. In contrast, since a cash offer's value is independent of $s_0$, a target would not need to consult its private information in deciding whether to accept or reject it. Thus, a cash offer cannot induce the target to make an efficient accept/reject decision.\footnote{Bidder 1 receives zero with certainty from such an offer. This assumption eliminates the strategy of never dropping out of the bidding. A rigorous way to exclude such offers is to assume a cost (no matter how small) either of bidding or of defaulting on debt.}

\footnote{The cash and debt offers are equivalent to offers explicitly contingent on $s_0$. An offer with $p(s_0) \geq v_0$ for $s_0 = \alpha$, $\beta$, is equivalent to a cash offer, and an offer with $p(s_0) < v_0 = p(\beta)$ is equivalent to a debt offer. In practice, securities offers commonly include equity of the merged entity. As the model stands, the debt offer dominates any offer that includes equity. This is because equity induces an adverse selection problem. Suppose, though, it is assumed that (i) bidders have no private information on the value of their assets in place and (ii) after bidder 2 makes its entry decision, $s_1$ and $s_2$ are observable by all. Then the equilibrium outcome to be derived can be obtained with cash and mixed cash/equity offers. The assumptions ease the adverse selection problem. In Hansen [12], securities offers consist of equity of the merged entity. This, however, is by assumption. The optimal securities offer there is also one backed only by target assets. Further, if allowed, such offers would dominate cash, and, thus, cash offers would not be observed.}
Let \((p, \theta)\) denote bidder 1's initial offer, where \(p \in [v_0, v^b(s_1)]\) is the face value and \(\theta \in [C, D]\) is the medium of exchange—cash or debt, respectively. Let \(e\) denote bidder 2's decision, where \(e = 1\) (0) denotes competing (not competing).

Finally, let \(\pi_i(s_1, s_2, s_0, p, \theta, e)\) denote bidder \(i\)'s payoff as a function of the specified arguments. We will first state and then explain the payoffs for the case when bidder 1 studies the target and bids. Define \(l_0\) such that \(v^d(l_0) = v_0\). Since the reservation price is \(v_0\), bidder 1 only bids if \(s_1 \geq l_0\). For \(s_1 = l_0\), bidder 1's payoffs, gross of the sunk cost \(k_0\), are as follows. If bidder 1 makes an initial debt offer \((p, D)\), its payoffs are given by

\[
\pi_1(s_1, s_2, s_0, p, D, 0) = \begin{cases} 0, & \text{if } s_0 = \alpha; \\ v^d(s_1) - p, & \text{if } s_0 = \beta; \end{cases}
\]

If bidder 1 makes an initial cash offer, \((p, C)\), its payoffs are given by

\[
\pi_1(s_1, s_2, s_0, p, C, 0) = \begin{cases} v^c(s_1) - p, & \text{if } s_0 = \alpha; \\ v^d(s_1) - p, & \text{if } s_0 = \beta; \end{cases}
\]

If bidder 1 makes an initial offer, \((p, \theta)\), bidder 2's payoffs are given by

\[
\pi_2(s_1, s_2, s_0, p, \theta, 0) = 0;
\]

\[
\pi_2(s_1, s_2, s_0, p, \theta, 1) = \begin{cases} -b_2, & \text{if } s_0 = \alpha; \\ v^d(s_2) - \min\{v^d(s_1), v^d(s_2)\} - k_2, & \text{if } s_0 = \beta. \end{cases}
\]

Say bidder 1 initially offers \((p, D)\). If bidder 2 competes, an auction follows. Note that, once in an auction, there is no incentive to offer cash since debt can promise the same payment as cash if \(s_0 = \beta\) but can also promise a low payment if \(s_0 = \alpha\). The face value of the ultimately highest valued offer is determined as follows. If \(v^d(s_2) \leq p\), bidder 2 will not top the initial bid. If \(v^d(s_2) > p\), the high bid is bid up to the second highest valuation. So the high bid is a debt offer with face value \(\min\{\max(p, v^d(s_2)), v^d(s_1)\}\) if bidder 2 competes and is the initial offer \((p, D)\) otherwise. Whichever it is, the target accepts (rejects) it if \(s_0 = \beta\) \((s_0 = \alpha)\).

Say bidder 1 initially offers \((p, C)\). If bidder 2 does not compete, the target accepts this offer regardless of \(s_0\). If bidder 2 competes, an auction follows, and, if \(s_0 = \beta\), the payoffs are the same as with an initial debt offer. If, however, \(s_0 = \alpha\), the target has no incentive to await the outcome of an auction, for, as discussed above, once in an auction, all bids consist of debt; so, if \(s_0 = \alpha\), the target accepts the initial cash offer.

Notice that bidder 2's expected payoff from competing is decreasing in \(s_1\). Also notice that, for a given decision as to whether or not to compete, bidder 2's payoff does not depend on bidder 1's initial offer. In making this decision, however, bidder 2 may find bidder 1's offer to be important as it may convey information on \(s_1\).
Two final assumptions are made. Assume that

\[ E[\pi_2(\bar{s}_1, \tilde{s}_2, \tilde{s}_0, p, \theta, 1) \mid \tilde{s}_1 \geq l_0] > 0. \]  

(3)

The minimum learned from bidder 1’s initial bid is that \( s_1 \geq l_0 \). Assumption (3) posits that this information is not sufficient to deter bidder 2. Define \( r \) such that \( E[\pi_2(\bar{s}_1, \tilde{s}_2, \tilde{s}_0, p, \theta, 1) \mid \tilde{s}_1 > r] = 0 \); \( r \) is the minimum value for which, if bidder 2 knew that \( s_1 \), exceeded this value, bidder 2 would not find it profitable to compete. Assumption (3) implies that \( l_0 < r < h \). Also assume that

\[ E\pi_1(r, \tilde{s}_2, \tilde{s}_0, v_0, C, 0) = E\pi_1(r, \tilde{s}_2, \tilde{s}_0, v_0, D, 1). \]  

(4)

As will be seen, (4) implies that a cash offer equal to \( v_0 \) is not sufficient to deter bidder 2. Deterrence requires a positive premium bid. (Relaxation of (3) and (4) is discussed in footnote 9.)

Before proceeding, a few comments are in order. The target value if acquired depends both on factors specific to the target, i.e., \( s_0 \), and factors specific to the bidder, i.e., \( s_1 \) or \( s_2 \). Target-specific factors may represent the condition of the target’s physical assets, and bidder-specific factors may represent the bidder’s talent in managing resources in the target’s industry. It has been assumed that \( \tilde{s}_1 \) and \( \tilde{s}_2 \) are independent. This implies (see (2)) that bidder 2’s expected payoff from competing is decreasing in \( s_1 \). It is more profitable to compete against a less talented bidder since a less talented bidder cannot afford to bid as much. This is a necessary condition for the results that follow, though independence of \( \tilde{s}_1 \) and \( \tilde{s}_2 \) is not necessary. If this condition does not hold, bidder 1 has no incentive to signal that it has a high valuation for the target. Such information would not be a deterrent. Consider also the assumption that target management has private information on each bidder’s valuation but not on target value in the absence of a change in control. This allows for a known target reservation price. One interpretation is as follows. As with bidders, suppose that target value under current management depends on factors specific to itself and factors specific to the target assets. Suppose also that the target cash flow history is observable. Then, target value under current management may be observable, even though the factors specific to current management and assets are not.

Pure strategy equilibria are considered. Denote bidder 1’s strategy as \( \sigma_1 = (\sigma_p, \sigma_s) \); \( \sigma_p \) and \( \sigma_s \) specify the face value and medium of exchange, respectively, of bidder 1’s initial offer, each as a function of \( s_1 \). Denote bidder 2’s strategy as \( \sigma_2 \); \( \sigma_2 \) specifies bidder 2’s response as a function of bidder 1’s initial offer. Let the density function \( g(s_1 \mid p, \theta) \) denote bidder 2’s updated beliefs on \( s_1 \), conditional on bidder 1’s initial offer.

The definition of equilibrium is based on the concept of Perfect Sequential Equilibrium (PSE).\(^5\) Let \( S(p, \theta; \sigma_1) = \{s_1 \mid g(s_1 \mid s_1 = p, \theta) \} \); \( S \) denotes the set of \( s_1 \) for which bidder 1 would make a given offer when using a particular strategy. For a nonempty \( S \subset [l_0, h] \), let

\[
\hat{g}(s_1; S) = \begin{cases} 
\frac{f_1(s_1)}{\int_{s_1 \in S} f_1(s_1) \, ds_1} & \text{if } s_1 \in S, \\
0 & \text{otherwise.}
\end{cases}
\]

Medium of Exchange in Acquisitions

**Definition:** The triple \((\sigma_1, \sigma_2, g)\) constitutes a PSE if and only if:

(Sequentially Rational Strategies) taking \(g\) as given,

(i) \(\sigma_1\) maximizes bidder 1's expected payoff given \(\sigma_2\),

(ii) for all \((p, \theta)\), \(\sigma_2\) maximizes bidder 2's expected payoff;

(Credible Beliefs) taking \((\sigma_1, \sigma_2)\) as given, for all \((p, \theta)\),

(i) if \(\hat{S}(p, \theta; \sigma_1)\) is nonempty, then \(g(s_1 | p, \theta) = \hat{g}(s_1 ; \hat{S}(p, \theta; \sigma_1))\),

(ii) if \(\hat{S}(p, \theta; \sigma_1)\) is empty, and if there is a unique nonempty set \(S \subset [b_0, h]\) such that \(E_{x_1}(s_1, \hat{s}_2, \hat{s}_0, p, \theta, \hat{\tau}(S)) > E_{x_1}(s_1, \hat{s}_2, s_0, \sigma_2(s_1), \sigma_4(s_1), \sigma_2(\sigma_2(s_1)), \sigma_4(s_1))\) if and only if \(s_1 \in S\), where \(\hat{\tau}(S)\) is an optimal response for bidder 2 given beliefs \(\hat{g}(s_1 | S)\), then \(g(s_1 | p, \theta) = \hat{g}(s_1 ; S)\).

Conditions (5) and (6ii) are based on the concept of Sequential Equilibrium. Bidders must always follow an optimal strategy given beliefs, and, in equilibrium, bidder 2's beliefs must be consistent with bidder 1's strategy and Bayes' rule. Condition (6ii), the additional condition required by a PSE, restricts the extent to which beliefs can be formed in response to out-of-equilibrium offers. Suppose that, for an out-of-equilibrium offer, \((p, \theta)\), there is a set \(S\) that satisfies (6ii). Then, first bidders for which \(s_1 \in S\), and only those first bidders, will prefer to deviate from the proposed strategy and offer \((p, \theta)\) if it will induce bidder 2 to believe \(s_1 \in S\). In such a case, credibility requires bidder 2 to believe \(s_1 \in S\).

**II. Analysis of Equilibrium**

Bidder 1, in making its initial offer, takes into account the effect of the offer on the beliefs of bidder 2, for, if bidder 2 believes bidder 1 has a sufficiently high valuation, it chooses not to compete. This is the strategic interaction characterized in equilibrium. Sequentially rational strategies for a fixed updating rule (for bidder 2) are derived first. Then, a credible updating rule given sequentially rational strategies is characterized. Combining these results yields the unique PSE outcome.

Upon observing bidder 1's offer, bidder 2 updates its beliefs and computes its expected payoff from competing. Bidder 2's optimal strategy is to compete if this expected payoff is positive and not to compete otherwise. For an updating rule \(g\), and \(\theta \in [C, D]\), let \(Q(\theta; g)\) denote the set of \(p\) for which the offer \((p, \theta)\) induces bidder 2 to expect a nonpositive payoff from competing, and suppose \(Q(\theta; g)\) is nonempty. Let \(q(\theta; g)\) equal the minimum element of \(Q(\theta; g)\); \(q(C; g)\) and \(q(D; g)\) represent the minimum cash and debt offers that deter bidder 2. Bidder 1's optimal strategy is to offer either \((q(D; g), D)\), \((q(C; g), C)\), or \((v_0, D)\)—either a (minimum) preemptive offer or a zero premium debt offer (i.e., an offer of the

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5 See Kreps and Wilson [15].

6 If \(Q(\theta; g)\) contains no minimum element, let \(q(\theta; g) = \inf(Q(\theta; g)) + \epsilon\), where \(\epsilon\) is positive and very small.
reservation value, contingent on a profitable acquisition). All other offers are
dominated.

To determine which first bidders make which offer, the expected payoffs from
these three offers must be compared. First compare preemptive with nonpreem-
tive offers. For \( s_1 \in [l_0, h] \), define

\[
p_D(s_1) = E \min \{ \max (u_0, u^\theta(\hat{s}_2)), u^\theta(s_1) \},
\]

\[
p_C(s_1) = \gamma E \min \{ \max (u_0, u^\theta(\hat{s}_2)), u^\theta(s_1) \} + (1 - \gamma) u^\theta(s_1),
\]

and let \( s_D(\cdot) \) and \( s_C(\cdot) \) denote the respective inverse functions. Using (1), it can
be verified that, for \( \theta \in [C, D] \), bidder 1’s expected payoff from the preemptive
offer \((q(\theta; g), \theta)\) exceeds the expected payoff from the offer \((\omega_0, D)\) if \( q(\theta; g) < p_D(s_1) \)
or, equivalently, if \( s_0(q(\theta; g)) < s_1 \). High valuing bidders make preemptive
offers. The reason is as follows. For a given offer \((p, \theta)\), the value to bidder 1 of
deterring bidder 2 equals

\[
E \pi_1(s_1, \hat{s}_2, \hat{s}_0, p, \theta, 0) - E \pi_1(s_1, \hat{s}_2, \hat{s}_0, p, \theta, 1)
= \gamma [ E \min \{ \max (p, u^\theta(\hat{s}_2)), u^\theta(s_1) \} - p ],
\]

the difference between bidder 1’s expected payoff if bidder 2 is deterred and
bidder 1’s expected payoff if bidder 2 is not deterred. This difference is increasing
in \( s_1 \). Higher valuing first bidders stand to lose more in an auction and thus have
a greater incentive to deter competition.

Now compare the two preemptive offers. Let \( z(s_1, p^D, p^C) = \gamma p^D + (1 - \gamma) u^\theta(s_1) - p^C \), and define \( s_{DC}(p^D, p^C) \) as follows. If there is an \( s \in [l_0, h] \) such
that \( z(s, p^D, p^C) = 0 \), then \( s_{DC}(p^D, p^C) = s \). Otherwise, if \( z(l_0, p^D, p^C) < 0 \), then
\( s_{DC}(p^D, p^C) = l_0 \), and, if \( z(h, p^D, p^C) < 0 \), then \( s_{DC}(p^D, p^C) = h \). Using (1), it can
be verified that bidder 1’s expected payoff is higher with the preemptive cash
(debt) offer if \( s_1 > (\leq) s_{DC}(q(D; g), q(C; g)) \). Of the bidders that make preemptive
bids, the high-valuing ones use cash and the low-valuing ones use debt. The
reason is as follows. A debt offer is only accepted if \( s_0 = \beta \), while a cash offer
is always accepted. Thus, a cash offer is a commitment to acquire the target if \( s_0 =
\alpha \), that is, even when the acquisition is unprofitable. For an offer with a given
face value \( p \), the expected cost of this commitment equals

\[
E \pi_1(s_1, \hat{s}_2, \hat{s}_0, p, D, e) - E \pi_1(s_1, \hat{s}_2, \hat{s}_0, p, C, e) = (1 - \gamma) [p - u^\theta(s_1)],
\]

the difference between bidder 1’s expected payoff if it offers debt and bidder 1’s
expected payoff if it offers cash. This difference is decreasing in \( s_1 \). The commit-
ment is cheaper for higher valuing bidders, and, for given preemptive offers, if
any bidders use cash it is the higher valuing ones.

These results on optimal strategies are summarized in Lemma 1.8

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8The following conventions are adopted. If bidder 1 is indifferent between a preemptive and
nonpreemptive offer, it makes a nonpreemptive offer, and, if it is indifferent between a preemptive
cash offer and a preemptive debt offer, it makes a preemptive debt offer. If bidder 2 is indifferent:
between competing and not competing, it does not compete. Also, if \( Q(\theta; g) \) is empty, let \( q(\theta; g) = p_\theta(h) \) for \( \theta \in [C, D] \).
Lemma 1: For an updating rule $g$, the strategies that satisfy (5) are

\begin{align}
\sigma_1(s_1) &= \begin{cases} 
(w_0, D) & \text{if } l_0 \leq s_1 \leq \min\{s_D(q(D; g)), s_C(q(C; g))\}, \\
(q(D; g), D) & \text{if } s_D(q(D; g)) < s_1 \leq s_{DC}(q(D; g), q(C; g)), \\
(q(C; g), C) & \text{if } s_1 > \max\{s_C(q(C; g)), s_{DC}(q(D; g), q(C; g))\};
\end{cases} \\
\sigma_2(p, \theta) &= \begin{cases} 
1 & \text{if } E[\pi_2(s_1, \tilde{s}_1, \tilde{s}_0, p, \theta, 1)|p, \theta] > 0, \\
0 & \text{if } E[\pi_2(s_1, \tilde{s}_1, \tilde{s}_0, p, \theta, 1)|p, \theta] \leq 0.
\end{cases}
\end{align}

A credible updating rule given such strategies is now characterized. Recall that $r$ is defined such that $E[\pi_2(s_1, \tilde{s}_1, \tilde{s}_0, p, \theta, 1)|\tilde{s}_1 > r] = 0$.

Lemma 2: If $(\sigma_1, \sigma_2, g)$ constitutes a PSE, then (i) $q(D; g) \geq p_D(r)$ and (ii) $q(C; g) = p_C(r)$.

Proof: See the Appendix.

In equilibrium, preemptive offers must be sufficiently high so that only first bidders with valuations high enough to deter second bidders have the incentive to make them. No debt offer below $p_D(r)$ preempts bidder 2, and the minimum preemptive cash offer is $p_C(r)$. Notice that, if $\gamma = 1$, the target effectively possesses no private information. In this case, $p_D(r) = p_C(r)$ since cash is then equivalent to debt. If $\gamma < 1$, the target does possess private information, and, in this case, $p_D(r) > p_C(r)$. The cash offer required to deter bidder 2 is below the face value of the debt offer required to do the same. As shown in Lemma 1, for any given minimum preemptive bids, bidders with higher values of $s_1$ are the ones that find it profitable to preempt with cash. Thus, the face value needed to signal that $s_1$ is at or above any given level is lower with cash as compared to debt.

Combining the characterization of sequentially rational strategies with the characterization of credible beliefs yields Proposition 1.

Proposition 1:
(i) There exists a PSE.
(ii) If $(\sigma_1^*, \sigma_2^*, g^*)$ constitutes a PSE, then

\begin{align}
\sigma_1^*(s_1) &= \begin{cases} 
(w_0, D) & \text{if } l_0 \leq s_1 \leq r, \\
(p_C(r), C) & \text{if } s_1 > r;
\end{cases} \\
\sigma_2^*(p, \theta) &= \begin{cases} 
1 & \text{if } p < p_D(r) \text{ and } \theta = D, \\
1 & \text{if } p < p_C(r) \text{ and } \theta = C, \\
0 & \text{if } p = p_C(r) \text{ and } \theta = C.
\end{cases}
\end{align}

Proof: See the Appendix.

In equilibrium, low-valuing bidders offer debt and high-valuing bidders offer cash. The information signaled by the cash offer preempts bidder 2, while the
information signaled by the debt offer does not.\footnote{In a Sequential Equilibrium, Lemmas 1 and 2(i) are unchanged. However, a Sequential Equilibrium only implies that $q(C; g) \geq p_C(r)$. This weaker result implies the existence of a continuum of Sequential Equilibrium outcomes characterized by the particular values of $q(C; g)$ and $q(D; g)$.

Say (3) is not satisfied. In equilibrium, if $s_i = b_i$, bidder 1 offers $(v_0, D)$, which preempts bidder 2. Say (3) is satisfied, but (4) is not. Noting that (4) is equivalent to $p_C(r) \geq v_0$, say $p_C(r) \leq v_0 < p_C(h)$. In equilibrium, low-valuing first bidders offer $(v_0, D)$ and bidder 2 competes, and high-valuing first bidders offer $(v_0, C)$, which preempts bidder 2. There may also be a range of intermediate valuations for which bidder 1 offers $(p, D)$, where $p > p_C(r)$, which also preempts bidder 2. Say $p_C(h) < v_0$. In equilibrium, if $b_1 = b_1 < r$, bidder 1 offers $(v_0, D)$ and bidder 2 competes, and, if $b_1 > r$, bidder 1 offers $(p_0(r), D)$, which preempts bidder 2.

The analysis suggests some interesting implications for contract theory. An important question is why certain contracts are (seemingly) "incomplete" in that there appear to be relevant and observable variables that are not contracted upon. The results here suggest that incomplete contracts may be agreed upon in an attempt to signal private information.} Note that, as $\gamma$, the probability that the acquisition is profitable (i.e., $s_0 = \alpha$), increases, bidder 2 becomes more difficult to deter and there are fewer preemptive bids ($r$, the threshold between preempting and nonpreempting bidders, is increasing in $\gamma$). As a result, as $\gamma$ increases, even though securities offers afford protection against the increasingly likely event of an unprofitable acquisition (i.e., $s_0 = \alpha$), they are made more frequently.\footnote{The analysis suggests some interesting implications for contract theory. An important question is why certain contracts are (seemingly) "incomplete" in that there appear to be relevant and observable variables that are not contracted upon. The results here suggest that incomplete contracts may be agreed upon in an attempt to signal private information.}

Using the equilibrium strategies from Proposition 1, equilibrium expected payoffs can be computed. Bidder 1's equilibrium expected payoff from studying the target equals (note that, if bidder 2 competes, $p_D(s_1)$ is the expected acquisition price for a given $s_1$ and for $s_0 = \beta$)

$$E_{\pi_1}(\tilde{s}_1, \tilde{s}_2, \tilde{s}_0, \sigma^*(\tilde{s}_1), \sigma^*(\tilde{s}_1), \sigma^*(\tilde{s}_1), s^*(\tilde{s}_1), s^*(\tilde{s}_1)) - k_1 = E[\gamma(u^0(\tilde{s}_1) - p_D(\tilde{s}_1)) \mid l_0 \leq \tilde{s}_1 \leq r] pr(l_0 \leq \tilde{s}_1 \leq r)$$

$$+ E[\gamma u^0(\tilde{s}_1) + (1 - \gamma)u^0(\tilde{s}_1) - p_C(r) \mid \tilde{s}_1 > r] pr(\tilde{s}_1 > r) - k_1$$

$$= \gamma E[u^0(\tilde{s}_1) - p_D(\min(\tilde{s}_1, r)) \mid \tilde{s}_1 \geq l_0] pr(\tilde{s}_1 \geq l_0)$$

$$+ (1 - \gamma) E[u^0(\tilde{s}_1) - u^0(r) \mid \tilde{s}_1 > r] pr(\tilde{s}_1 > r) - k_1.$$  (9)

Bidder 1 initially studies the target if (9) is positive. If this is the case, bidder 2's equilibrium expected payoff equals

$$E_{\pi_2}(\tilde{s}_1, \tilde{s}_2, \tilde{s}_0, \sigma^*(\tilde{s}_1), \sigma^*(\tilde{s}_1), \sigma^*(\tilde{s}_1), s^*(\tilde{s}_1), s^*(\tilde{s}_1)) = \gamma E[u^0(\tilde{s}_1) - \gamma \min(u^0(\tilde{s}_1), u^0(\tilde{s}_2)) - k_2 \mid l_0 \leq \tilde{s}_1 \leq r] pr(l_0 \leq \tilde{s}_1 \leq r),$$  (10)

and the target's equilibrium expected payoff equals

$$E[\tilde{s}_0] = v_0 pr(\tilde{s}_1 < l_0)$$

$$+ E[\gamma p_D(\tilde{s}_1) + (1 - \gamma)v_0 \mid l_0 \leq \tilde{s}_1 \leq r] pr(l_0 \leq \tilde{s}_1 \leq r) + p_C(r) pr(\tilde{s}_1 > r)$$

$$= v_0 pr(\tilde{s}_1 < l_0) + \gamma E[p_D(\min(\tilde{s}_1, r)) \mid \tilde{s}_1 \geq l_0] pr(\tilde{s}_1 \geq l_0)$$

$$+ (1 - \gamma)[v_0 pr(l_0 \leq \tilde{s}_1 \leq r) + u^0(r) pr(\tilde{s}_1 > r)].$$  (11)
Consider the implications of a decrease in bidder 2's cost of studying the target. Since bidder 2's cost of competing is lower, its expected payoff is higher. Bidder 1's expected payoff is lower since a higher valuation must be signalled to deter bidder 2, and this is more costly. In addition, bidder 1 is more likely to offer securities. Consider now the target. Bidder 2 is preempted less often, and, when it is preempted, the bid is higher. Both effects benefit the target, and its expected payoff is higher. Note that these results require bidder 1's expected payoff to remain positive. Otherwise no bidder will study the target initially.

Figure 1 illustrates the determination of \( r \), the critical value of \( s_1 \), that separates first bidders that offer cash \( (s_1 > r) \) from first bidders that offer securities \( (s_1 \leq r) \). In the example, \( \hat{s}_1 \) is uniformly distributed on \([0, 1] \), \( v_0 = 1 \), \( v^o(s_1) = 0.49 + s_1 \) (thus \( b_0 = 0.51 \), and \( v^c(s_1) = s_1/1.01 \). As is shown, \( r \) is decreasing in \( k_2 \), bidder 2's information cost, and increasing in \( \gamma \), the probability that the acquisition is profitable.

### III. Advance Disclosure of the Target's Information

It has been assumed that the target observes \( s_0 \), its private information on the profitability of an acquisition, after bidder 1 bids. Suppose, though, that the target observes and (truthfully) discloses \( s_0 \) prior to bidder 1's initial offer. This section discusses the effects of such a disclosure. It is shown that bidder 1's expected payoff is decreased, and, provided that bidder 1 is not deterred, bidder 2's and the target's expected payoffs are increased.

If the target discloses \( s_0 \) after bidder 1 studies the target but before it bids, the

![Figure 1. Preemptive Bidding Examples. The critical value separating first bidders that offer cash (those with valuations above the critical value) from first bidders that offer securities (those with valuations below the critical value), as a function of bidder 2's information cost, is shown. \( \gamma \) denotes the probability that the acquisition is profitable.](image-url)
nature of the bidding problem is unchanged, with the exception that \( s_0 \) is known. With probability \( 1 - \gamma \), \( s_0 = \alpha \) is disclosed, and there is no bidding. Bidder 1's payoff is \(-h_1\), bidder 2's payoff is zero, and the target’s payoff is \( v_0\). With probability \( \gamma \), \( s_0 = \beta \) is disclosed, and the expected payoffs for bidders 1 and 2 and the target are given by (9), (10), and (11), respectively, evaluated at \( \gamma = 1 \). Define \( r' \) such that \( E[\pi_2(\hat{s}_1, \hat{s}_2, \beta, \theta, \theta, \theta)] \mid \hat{s}_i > r'] = 0 \); if it is known that \( s_0 = \beta \), then \( r' \) is the minimum value for which, if bidder 2 knew that \( s_0 \) exceeded this value, bidder 2 would not find it profitable to compete. If \( \gamma < 1 \), then \( r' > r \). Advance disclosure of \( s_0 = \beta \) makes bidder 2 more difficult to deter.

Bidder 1's expected payoff with advance disclosure equals

\[
E[\hat{z}_{0,2}'] - h_1 = \gamma E[u^D(\hat{s}_1) - p_D(\min(\hat{s}_1, r'))] \mid \hat{s}_i \geq l_0] pr(\hat{s}_i \geq l_0) - h_1. \tag{12}
\]

If \( \gamma < 1 \), (9) exceeds (12). That is, bidder 1's expected payoff is lower with advance disclosure. There are two reasons. The first is that bidder 2 observes the target's information prior to deciding whether to compete, and this makes competing more profitable for bidder 2. Thus, preemption by bidder 1 now requires signaling a higher valuation, and this is more costly. The second reason is that bidder 1 observes the target's information prior to its own bidding. Since bidder 1 can make a debt offer, which is contingent on the target's information, observing this information prior to bidding yields no benefits. Further, the benefit of a noncontingent cash offer is eliminated. Cash establishes a commitment to acquire the target even if it turns out that the target’s information indicates an unprofitable acquisition. In equilibrium, high-valuing bidders find it profitable to establish this commitment in order to signal. With advance disclosure of the target's information, the cost of such a commitment is zero, and the efficacy of cash in signaling is eliminated. This lowers bidder 1's expected payoff.

If bidder 1 will still study the target initially, then bidder 2's expected payoff with advance disclosure equals

\[
E[\hat{z}_{0,2}''] = \gamma E[v^D(\hat{s}_1) - \min\{v^D(\hat{s}_1), v^D(\hat{s}_2)\} - h_2 \mid l_0 \leq \hat{s}_1 \leq r'] pr(l_0 \leq \hat{s}_1 \leq r'), \tag{13}
\]

and the target’s expected payoff with advance disclosure equals

\[
E[\hat{z}_{0,2}'''] = \gamma [u_0 pr(\hat{s}_1 < l_0) + E[p_D(\min(\hat{s}_1, r'))] \mid \hat{s}_i \geq l_0] pr(\hat{s}_i \geq l_0)] + (1 - \gamma) u_0. \tag{14}
\]

If \( \gamma < 1 \), (13) exceeds (10) and (14) exceeds (11). That is, both bidder 2's and the target's expected payoffs are higher with advance disclosure. Bidder 2's expected payoff is higher because the chance that it studies the target when the target's information indicates an unprofitable acquisition is eliminated. The target's expected payoff is higher because there are fewer preemptive bids, and any preemptive bids that do occur are higher.\footnote{\cite{MilgromWeber1982} present results of a similar nature in a study of auctions. It is shown that a seller with access to private information may raise its expected selling price by disclosing this information. Key to this result is that bidders' valuations are independent. \cite{MilgromWeber1982} and risk neutrality, disclosure has no effect on expected selling price. Here, even if there were independent valuations, the results would go through. Suppose that, instead of the target having private information common to both bidders' valuations, it has private information specific to each bidder. That is, suppose the target observes the independent random variables \( \tilde{s}_0 \) and \( \tilde{s}_0' \), and suppose \( \tilde{s}_0 \) conveys information only on bidder 1's valuation. If advance disclosure is taken to mean the disclosure of both \( \tilde{s}_0 \) and \( \tilde{s}_0' \), then the qualitative results of this section will be unchanged.}
This is an interesting rationale for firms to continually release information. It can make preemptive bids more costly and thus raise a firm's expected payoff in the event it becomes a candidate for acquisition. More generally, bidders will seek to lower the costs of preemption and targets will seek to raise these costs. An interesting question for future work concerns what other actions may derive from such incentives.

IV. Further Implications

An implication of the model is that target management is more likely to reject a securities offer as compared to a cash offer. Dodd [5] reports that, for a sample of merger offers rejected by target management, the target's share price dropped. One conclusion that might be drawn is that this is evidence of a firm's management acting contrary to the interests of its shareholders. The theory here suggests another possibility. Rejection of a securities offer indicates that the proposed acquisition is not profitable—-and thus the drop in share price. (The price will have risen at the offer's announcement.) It would have been worse for target shareholders, though, had the offer been accepted. It would be interesting to examine rejected offers conditional on the medium of exchange.12

The model also predicts that competing bidders are more likely to be observed following an initial securities offer as compared to an initial cash offer. This is because cash is only used in preemptive bids.13

Share price responses to acquisition announcements, conditional on the medium of exchange, can be considered. If $l_c \leq s_1 \leq r$, bidder 1 offers securities and faces competition. Thus, the market's expectation of bidder 1's payoff, conditional on its making a securities offer, equals (note that the market observes the initial offer, but not $s_1$)

$$\gamma E[u^d(\tilde{s}_1) - p_D(\tilde{s}_1) | l_c \leq \tilde{s}_1 \leq r] - k_1.$$  \hspace{1cm} (15)

If $s_1 > r$, bidder 1 offers cash and the competition is deterred. Thus, the market's expectation of bidder 1's payoff, conditional on its making a cash offer, equals

$$E[\gamma u^d(\tilde{s}_1) + (1 - \gamma)u^c(\tilde{s}_1) - p_C(r) | \tilde{s}_1 > r] - k_1.$$  \hspace{1cm} (16)

12 Travers [19] reports that, for a sample of merger offers and tender offers, the chance of success is lower if the medium of exchange is cash. It would be useful to examine the results for merger offers only. This is because the key to the model here is the assumption that the accept/reject decision is made by a party with private information on the target. This party is more reasonably interpreted as management rather than shareholders, and, thus, the model is best viewed as a model of merger offers.

13 Franks, Harris, and Mayer [8] report cases when initial bidders revise their bids but no competing bids are observed. A scenario in which the model would be consistent with this is one in which an initial bidder does not know whether there is a potentially competing bidder. Then it offers securities initially, and, if a competitor is observed, it may make a revised, preemptive cash offer. They also report that bidding contests in which the final offer is cash are more likely to have had revised bids or been met by competing bids than are contests in which the final offer includes securities. The likelihood of competing bids is not reported separately (from revised bids); nor is the likelihood as a function of the initial offer.
It can be verified that (16) exceeds (15). Thus, the share price revaluation for bidder 1 will be higher if it offers cash as compared to securities. Consistent with this prediction, Asquith, Bruner, and Mullins [1], Franks, Harris, and Mayer [8] (for the U.S., but not U.K.), Gordon and Yagil [10], and Travlos [19] report higher share price revaluations for bidders that made cash-only as compared to combination cash and securities merger offers.

Whether a target's share price revaluation is also higher given a cash offer as compared to a securities offer depends on the parameters of the model. Since there is competition following a securities offer, the market's expectation of a target's payoff given a securities offer equals

\[ \gamma E[p_D(s_i)|I_o \leq s_i \leq r] + (1 - \gamma)\tilde{\psi}_0. \]

Since there is no competition following a cash offer, a target's payoff from a cash offer is \( p_C(r) \). If the preemptive offer is high enough, a target's share price revaluation will be higher given a cash offer. Asquith, Bruner, and Mullins [1], Franks, Harris, and Mayer [8], Gordon and Yagil [10], Huang and Walking [13], and Wansley, Lane, and Yang [20] study target returns to merger offers, conditional on the medium of exchange. All report higher share price revaluations for targets of cash-only offers as compared to combination cash and securities offers.

As discussed in Section II, bidder 1's expected payoff decreases and the target's expected payoff increases, with a decrease in \( h_2 \), bidder 2's information cost. Consider the effect on share price responses to acquisition announcements, conditional on the medium of exchange. It follows from (15) that bidder 1's share price revaluation, conditional on its offering securities, increases if \( h_2 \) decreases. This is because \( r \), the critical value of \( s_i \) that separates first bidders that offer cash from those that offer securities, is decreasing in \( h_2 \). That is, first bidders that offer securities now have a higher average valuation. Using (16), it can be shown that bidder 1's share price revaluation, conditional on its offering cash, may also increase if \( h_2 \) decreases. Though the preemptive bid is higher, first bidders that offer cash also have a higher average valuation, so, while a first bidder's expected payoff decreases if \( h_2 \) decreases, share price revaluations for a first bidder, conditional on securities offers and cash offers, may both increase. For the target, share price revaluations, conditional on both cash and securities offers, increase if \( h_2 \) decreases.

These implications were derived in a model based on informational asymmetries. A prominent competing model is tax based; bidders structure offers to minimize the total tax liability of bidder and target shareholders. Such a model also predicts higher premiums for cash offers as compared to securities offers. Target shareholders must realize any capital gain from a cash offer immediately, whereas a capital gain from a securities offer can be deferred until the securities are sold. Thus, for a given after-tax reservation price, target shareholders require a higher premium to sell their shares for cash. Franks, Harris, and Mayer [8], however, report higher premiums for cash offers in the U.K. for a time period before the introduction of a capital gains tax. Thus, while tax considerations may

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help in understanding the choice of medium of exchange, they do not provide the complete explanation.

**Appendix**

**Proof of Lemma 2:** Let \( \Pi_i(s_i, p, \theta, e) = E_x(s_i, \hat{s}_i, \bar{s}_i, p, \theta, e) \), and, for a nonempty set \( S \subseteq [c_i, h] \), let \( w(S) = E_x(s_i, \hat{s}_i, \bar{s}_i, p, \theta, 1) \mid \hat{s}_i \in S \).

(i) Establish \( q(D; g) \geq p_D(r) \).

Say \( q(D; g) < p_D(r) \). There are two cases to consider.

(a) \( s_{DC}(q(D; g), q(C; g)) \leq s_D(p_D(r)) = r \).

In this case, \( s_{DC}(q(D; g), q(C; g)) \leq s_D(q(D; g)) < s_D(p_D(r)) = r \).

Therefore, \( \gamma p_D(r) + (1 - \gamma) q(r) - q(C; g) > \gamma q(D; g) + (1 - \gamma) q(C; g) \geq 0 \).

This implies \( q(C; g) < p_C(r) \), which implies \( s_C(q(C; g)) < s_C(p_C(r)) \).

Using Lemma 1, \( \hat{S}(q(C; g), C; \sigma_i) = (\max[s_C(q(C; g))], s_{DC}(q(D; g), q(C; g))], h) \), and (6i) requires \( g(s_1 \mid q(C; g), C) = \hat{g}(s_1; \hat{S}(q(C; g), C; \sigma_i)) \).

However, \( w((\max[s_C(q(C; g))], s_{DC}(q(D; g), q(C; g))], h)) > w((r, h)) = 0 \), and, thus, \( q(C; g) \notin Q(C; g) \), which is a contradiction.

(b) \( s_{DC}(q(D; g), q(C; g)) > s_D(p_D(r)) \).

Using Lemma 1, \( \hat{S}(q(D; g), D; \sigma_i) = (s_D(q(D; g)), s_{DC}(q(D; g), q(C; g))], h) \) and (6i) requires \( g(s_1 \mid q(D; g), D) = \hat{g}(s_1; \hat{S}(q(D; g), D; \sigma_i)) \).

However, \( w((s_D(q(D; g)), s_{DC}(q(D; g), q(C; g))], h)) > w((s_D(p_D(r)), h)) = w((r, h)) = 0 \), and, thus, \( q(D; g) \notin Q(D; g) \), which is a contradiction.

Thus, \( q(D; g) \geq p_D(r) \).

(ii) Establish \( q(C; g) = p_C(r) \).

Say \( q(C; g) < p_C(r) \).

Then, \( s_C(q(C; g)) < s_C(p_C(r)) = r \) and \( s_{DC}(q(D; g), q(C; g)) < s_{DC}(p_D(r), p_C(r)) = r \).

Using Lemma 1, \( \hat{S}(q(C; g), C; \sigma_i) = (\max[s_C(q(C; g))], s_{DC}(q(D; g), q(C; g))], h) \) and (6i) requires \( g(s_1 \mid q(C; g), C) = \hat{g}(s_1; \hat{S}(q(C; g), C; \sigma_i)) \).

However, \( w((\max[s_C(q(C; g))], s_{DC}(q(D; g), q(C; g))], h)) > w((r, h)) = 0 \), and, thus, \( q(C; g) \notin Q(C; g) \), which is a contradiction.

Say \( q(C; g) = p_C(r) \).

Using Lemma 1, \( \hat{S}(p_C(r), C; \sigma_i) \) is empty. It will be shown that there exists an \( S \) that satisfies (6ii) for the offer \( (p_C(r), C) \). There are two possibilities to consider.

(a) Say \( S \subseteq [c_i, h] \) and \( w(S) > 0 \).

For \( s_1 \subseteq [c_i, h] \), \( \Pi_i(s_i, p_C(r), C, \delta(S)) = \Pi_i(s_i, p_C(r), C, 1) < \Pi_i(s_i, w_0, D, 1) \leq \Pi_i(s_i, s_0, s_1, s_2, s_3, s_4) \) and \( \Pi_i(s_i, s_0, s_1, s_2, s_3, s_4) \), and there exists no such \( S \) that satisfies (6ii).

(b) Say \( S \subseteq [c_i, h] \) and \( w(S) \leq 0 \).

For \( s_1 \subseteq [c_i, h] \), \( \Pi_i(s_1, p_C(r), C, \delta(S)) = \Pi_i(s_1, p_C(r), C, 0) > \Pi_i(s_1, q(C; g), C, 0) = \Pi_i(s_1, q(C; g), C, \sigma_C(q(C; g), C)) \).
Using $s_c(p_C(r)) = r$, for $s_1 \in (r, h]$, $\Pi_1(s_1, p_C(r), C, \hat{\delta}_2(S)) = \Pi_1(s_1, p_C(r), C, 0) > \Pi_1(s_1, v_0, D, 1) = \Pi_1(s_1, v_0, D, \sigma_2(v_0, D))$.

For $s_1 \in (s_2(p_C(r)), h]$, $\Pi_1(s_1, p_C(r), C, \hat{\delta}_2(S)) = \Pi_1(s_1, p_C(r), C, 0) > \Pi_1(s_1, q(D; g), D, 0) = \Pi_1(s_1, q(D; g), D, \sigma_2(q(D; g), D))$.

Thus, since $\sigma_2(s_1)$ equals $(v_0, D)$, $(q(D; g), D)$, or $(q(C; g), C)$, we have that, for $s_1 \in [(b_1, h] \cap (r, h] \cap (s_{DC}(q(D; g), p_C(r)), h], \Pi_1(s_1, p_C(r), C, \hat{\delta}_2(S)) > \Pi_1(s_1, \sigma_2(s_1), \sigma_2(s_1)), \sigma_2(s_1))$.

Lemma 2(ii) implies $s_{DC}(q(D; g), p_C(r)) \leq s_{DC}(p_D(r), p_C(r)) = r$.

Therefore, $(b_1, h] \cap (r, h) \cap (s_{DC}(q(D; g), p_C(r)), h] = (r, h]$.

Using $s_{DC}(p_C(r)) = r$, for $s_1 \in [b_1, r]$, $\Pi_1(s_1, p_C(r), C, \hat{\delta}_2(S)) = \Pi_1(s_1, p_C(r), C, 0) \leq \Pi_1(s_1, v_0, D, 1) = \Pi_1(s_1, v_0, D, \sigma_2(v_0, D)) \leq \Pi_1(s_1, \sigma_2(s_1), \sigma_2(s_1), \sigma_2(s_1))$.

Finally, $w(r, h] = 0$, and we have that $S = (r, h]$ satisfies (6ii).

Therefore, $g(s_1 | p_C(r), C) = \hat{g}(s_1; r, h]$.

However, $w(r, h] = 0$, and thus, $p_C(r) \in Q(C; g)$, which is a contradiction.

Thus, $q(C; g) = p_C(r)$. Q.E.D.

Proof of Proposition 1:

(i) It can be verified that the updating rule

$$g(s_i \mid p_i, \theta) = \begin{cases} \hat{g}(s_i; [b_1, r]) & \text{if } \theta = D, \\ \hat{g}(s_i; (s_C(p), h]) & \text{if } \theta = C, \end{cases}$$

combined with strategies given by (8), constitutes a PSE.

(ii) Suppose $(\sigma^*_1, \sigma^*_2, g^*)$ constitutes a PSE.

That $\sigma^*_1$ satisfies (8b) follows directly from Lemma 2.

Using Lemma 2, $s_D(q(D; g)) > s_D(p_D(r)) = r$, $s_C(q(C; g)) = s_C(p_C(r)) = r$, and $s_{DC}(q(D; g), q(C; g)) \leq s_{DC}(p_D(r), p_C(r)) = r$.

Therefore, $\min(s_D(q(D; g)), s_C(q(C; g))) = r$, $s_D(q(D; g)) \geq s_{DC}(q(D; g), q(C; g))$, and $\max(s_D(q(D; g)), s_{DC}(q(D; g), q(C; g))) = r$.

That $\sigma^*_1$ satisfies (8a) then follows directly from Lemma 1. Q.E.D.

REFERENCES


